Total No. of Questions—**8**] [Total No. of Printed Pages—**7**

Seat No.	

[5057]-2011

S.E. (Mech./Mech/Auto/Sand etc.) (First Semester)

EXAMINATION, 2016

ENGINEERING MATHEMATICS

Paper III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt Q. **1** or Q. **2**, Q. **3** or Q. **4**, Q. **5** or Q. **6** and Q. **7** or Q. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^3 - 7D - 6)y = e^{2x}(1 + x)$

(*ii*) $(D^2 + 1)y = 3x - 8\cot x$ (by variation of parameter method).

(*iii*) $(2x + 1)^2 \dfrac{d^2y}{dx^2} - 2(2x + 1)\dfrac{dy}{dx} - 12y = 12x$.

(*b*) Find the Fourier cosine transform of : [4]

$$f(x) = e^{-x} + e^{-2x}, \ x > 0.$$

Or

2. (*a*) A body weighing W = 20 N is hung from a spring. A pull of 40 N will strech the spring to 10 cm. The body is pulled down to 20 cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position in time *t* seconds, the maximum velocity and period of oscillation. [4]

(*b*) Solve any *one* : [4]

(*i*) Find Laplace transform of :

$$t\,e^{-2t}(2\cosh 3t - 4\sinh 2t).$$

(*ii*) Find inverse Laplace transform of :

$$F(s) = \frac{s^2 - 2s + 3}{(s-1)^2\,(s+1)}.$$

(*c*) Using Laplace transform, solve the differential equation : [4]

$$\frac{dy}{dt} + 2y(t) + \int_0^t y(t)\,dt = \sin t$$

given $y(0) = 1$.

3. (*a*) The first four moments of a distribution about the value 2 are –2, 12, –20 and 100. Find the first four central moments and B_1, B_2. [4]

(*b*) Number of absent student in a class follow Poisson distribution with mean 5. Find the probability that in a certain month number of absent student in a class will be : [4]

(*i*) More than 3

(*ii*) Between 3 and 5.

(*c*) Find the directional derivative of $\phi = x^2 y z^3$ at (2, 1, –1) along the vector $-4\bar{i} - 4\bar{j} + 12\bar{k}$. [4]

Or

4. (*a*) Find coefficient of correlation for the following data : [4]

x	***y***
6	9
2	11
10	5
4	8
8	7

(*b*) Show that vector field : [4]

$$\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$$

is irrotational. Hence find scalar potential ϕ such that $\bar{F} = \nabla\phi$.

(*c*) Prove the following (any *one*) : [4]

(*i*) $\nabla^4 (r^2 \log r) = \dfrac{6}{r^2}$

(*ii*) $\bar{a} \cdot \nabla \left[\bar{b} \cdot \nabla\left(\dfrac{1}{r}\right)\right] = \dfrac{3(\bar{a}.\bar{r})(\bar{b}.\bar{r})}{r^5} - \dfrac{(\bar{a}.\bar{b})}{r^3}$

5. (*a*) Evaluate :

$$\int_c \overline{F}.d\overline{r}$$

where $\overline{F} = x^2\overline{i} + xy\overline{j}$ and '*c*' is the arc of the parabola joining (0, 0) and (1, 1). Equation of parabola is $y = x^2$. [4]

(*b*) Show that : [4]

$$\iint_s \frac{\overline{r}}{r^3} . \hat{n} \; ds = 0 .$$

(*c*) Verify Stoke's theorem for : [5]

$$\overline{F} = xy^2\,\overline{i} + y\overline{j} + z^2x\,\overline{k}$$

for the surface of a rectangular lamina, bounded by :

$$x = 0,\; y = 0,\; x = 1,\; y = 2,\; z = 0 .$$

Or

6. (*a*) Evaluate $\oint_c \overline{F}.d\overline{r}$ [4]

where $\overline{F} = (2x - y)\overline{i} + (x - 2y)\overline{j} + z\,\overline{k}$

and c is the arc of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1,\; z = 0$.

(*b*) Evaluate : [4]

$$\iint_s (x\bar{i} + y\bar{j} + z\bar{k}) \,.\, d\bar{s}$$

over the surface of a sphere $x^2 + y^2 + z^2 = 1$.

(*c*) Evaluate : [5]

$$\iint_s \text{curl}\, \bar{F} \,.\, \hat{n}\, ds$$

for the surface of the paraboloid $z = 9 - (x^2 + y^2)$ and

$$\bar{F} = (x^2 + y - 4)\,\bar{i} + 3xy\,\bar{j} + (2xz + z^2)\,\bar{k}.$$

7. Attempt any *two* :

(*a*) Solve the equation :

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

where $u(x, t)$ satisfies the following conditions : [6]

(*i*) $u(0, t) = 0$

(*ii*) $u(L, t) = 0$

(*iii*) $u(x, 0) = x,\ 0 \le x \le \frac{L}{2}$

$$= L - x,\ \frac{1}{2} \le x \le L$$

(*iv*) $u(x, \infty)$ is finite.

(*b*) If

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

represents the vibrations of a string of Length L fixed at both ends, find the solution with boundary conditions : [7]

(*i*) $y(0, t) = 0$

(*ii*) $y(\mathrm{L}, t) = 0$

(*iii*) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

(*iv*) $y(x, 0) = k(\mathrm{L}x - x^2), \ \ 0 \le x \le \mathrm{L}$.

(*c*) An infinitely long plane uniform plate is bounded by two parallel edges $x = 0$ and $x = \pi$ and an end at right angles to them. The breadth of the plate is π. This edge is maintained at temperature u_0 at all points and other edges at zero temperature. Find the steady state temperature function $u(x, y)$. Also use $y \to \infty, \ \ u = 0$. [6]

Or

8. Attempt any *two* :

(*a*) Solve : [6]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

if

(*i*) u is finite for all t

(*ii*) $u(0, t) = 0$

(*iii*) $u(\pi, t) = 0$

(*iv*) $u(x, 0) = \pi x - x^2, \ \ 0 \le x \le \pi$.

(*b*) A string streched and fastened to two points 'L' apart. Motion is started by displacing the string in the form $y = a\sin\frac{\pi x}{L}$ from which it is released at time $t = 0$. Find displacement $y(x, t)$ from one end. [7]

(*c*) A rectangular plate with insulated surface is 10 cm wide and so long to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of short edge $y = 0$ is given by : [6]

$$u = 20x, \quad 0 \le x \le 5$$

$$= 20(10 - x), \quad 5 \le x \le 10$$

and two edges $x = 0$ and $x = 10$ as well as other short edge are kept at 0ºC, then find $u(x, y)$.