

# 24209

**12223**

**3 Hours / 70 Marks**

Seat No. 

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- Instructions* –
- (1) All Questions are *Compulsory*.
  - (2) Answer each next main Question on a new page.
  - (3) Illustrate your answers with neat sketches wherever necessary.
  - (4) Figures to the right indicate full marks.
  - (5) Assume suitable data, if necessary.
  - (6) Use of Non-programmable Electronic Pocket Calculator is permissible.
  - (7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

**Marks**

**1. Attempt any FIVE of the following: **10****

- a) If  $f(x) = \log(\sin x)$ , find  $f\left(\frac{\pi}{2}\right)$
- b) Define odd and even functions.
- c) Differentiate with respect to  $x$ :  $\log_{10}^x + \log_{10}^{10} + 10^x + x^{10}$
- d) Evaluate  $\int \frac{dx}{3x^2+4}$
- e) Evaluate  $\int x \cdot \log x \, dx$
- f) Find area between the lines  $y = 2x$ ,  $x$ -axis and the ordinates,  $x = 1$  and  $x = 3$ .
- g) Show that root of equation  $x^3 - 9x + 1 = 0$  lies between 2 and 3.

P.T.O.

**2. Attempt any THREE of the following: 12**

- a) If  $f(x) = \frac{x-4}{4x-1}$  then show that  $f[f(x)] = x$ .
- b) If  $x^2 + 3xy + y^2 = 11$  find  $\frac{dy}{dx}$  at  $(1, 2)$ .
- c) If  $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$  and  $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$  find  $\frac{dy}{dx}$ .
- d) Find the maximum and minimum value of  $x^3 - 9x^2 + 24x$ .

**3. Attempt any THREE of the following: 12**

- a) An open box is to be made out of a rectangular sheet of metal measuring  $16 \text{ cm} \times 10 \text{ cm}$  by cutting off equal squares from the corners and turning up the sides. Find the side of the square if the volume of box is to be maximum.
- b) If  $e^y = y^x$  prove that  $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$
- c) If  $y = \log(x \sin 2x)$ . Find  $\frac{dy}{dx}$ .
- d) Evaluate  $\int \frac{5^{\tan x}}{\cos^2 x} dx$

**4. Attempt any THREE of the following: 12**

- a) Evaluate  $\int \frac{dx}{3-2x-x^2}$
- b) Evaluate  $\int \frac{dx}{5-\cos 2x}$
- c) Evaluate  $\int \frac{e^x}{(e^x-1)(e^x+1)} dx$
- d) Evaluate  $\int (\log x)^2 dx$
- e) Evaluate  $\int_0^4 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4-x}} dx$

5. Attempt any TWO of the following:

12

- a) Find area enclosed by two parabolas  $y^2 = 2x$  and  $x^2 = 2y$ .
- b) Attempt the following.
- i) An unbiased coin is tossed 3 times. Find probability of getting exactly two tails.
- ii) Fit a Poisson's distribution for the following observations.

Accident per shift ( $x_i$ )	0	1	2	3	4
Frequency ( $f_i$ )	192	100	24	3	1

- c) In a sample of 1000 students, mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal. Find how many student score.
- i) Between 12 and 15
- ii) Above 18.
- Given  $A(0.8) = 0.2881$   
 $A(0.4) = 0.1554$   
 $A(1.6) = 0.4452$

6. Attempt any TWO of the following:

12

- a) Attempt the following.
- i) Using Bisection method, find the approximate root of  $x^2 + x - 3 = 0$  (carry out two iterations).
- ii) Solve by Jacob's method (carry out two iterations).  
 $2x + y + z = 4$ ,  $x + 2y + z = 4$ ,  
 $x + y + 2z = 4$ .
- b) Solve the equation by Gauss Elimination method.  
 $4x + y + 2z = 12$ ,  $-x + 11y + 4z = 33$ ,  
 $2x - 3y + 8z = 20$ .
- c) Find the approximate value of  $\sqrt[3]{7}$  by using Newton Raphson method up to four iterations.

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