24209

22223

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Solve any FIVE of the following: **10**

a) Test whether the function is an even or odd.

$f(x) = x^4 + x\sin x - x^2 + \sin^2 x$.

b) If $f(x) = (x-1)(2x+1)$ then find $f(2)$ and $f(-3)$

c) Find $\frac{dy}{dx}$, if $y = x.\sin^{-1}x$

d) Evaluate $\int (x^a + a^x + a^a)dx$

e) Evaluate $\int \tan^{-1}x \; dx$

f) Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$.

g) Show that root of the equation $x^3 + 9x - 11 = 0$ lies between [1,2]

P.T.O.

Marks

2. Solve any THREE of the following: **12**

a) If $f(x) = \frac{x+5}{3x-4}$ and t = $\frac{5+4x}{3x-1}$ show that $f(t) = x$.

b) If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

c) If $y = \tan^{-1}\left[\frac{5x}{1-6x^2}\right]$, find $\frac{dy}{dx}$

d) Divide 30 into two parts such that product of one and cube of other is maximum.

3. Solve any THREE of the following: **12**

a) If $y = \sin^{-1}x$, then prove that $(1-x^2)\ \frac{d^2y}{dx^2} - x\ \frac{dy}{dx} = 0$

b) Find $\frac{dy}{dx}$, if $y = x^x + (\sin x)^x$

c) If $x^3 + y^3 = 3axy$ find $\frac{dy}{dx}$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

d) Evaluate $\int \frac{(\tan^{-1}x)^3}{1+x^2}dx$.

4. Solve any THREE of the following: **12**

a) Evaluate $\int \frac{1}{3+2\sin x}\ dx$.

b) Evaluate $\int \frac{e^x}{(e^x-1)(e^x+1)}\ dx$.

c) Evaluate $\int x^2.e^x\ dx$.

d) Evaluate $\int_0^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+4}}\ dx$

e) Evaluate $\int_0^4 \frac{dx}{\sqrt{4x-x^2}}$

Marks

5. Solve any TWO of the following: **12**

a) Find area bounded by two curves $y^2 = x$ and $x^2 = y$

b) Attempt the following

i) The probability that a student who is freshman will graduate is 0.4. Determine the probability that out of 5 students at least one will be graduate.

ii) If probability that an electric motor is defective is 0.01 what is probability that sample of 300 electric motor will contain exactly 5 defective motor ($\bar{e}^3 = 0.0498$)

c) In a sample of 1000 cases the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution is normal. Find

i) How many students score between 12 and 15.

ii) How many students score above 18
(Given $A(0.8) = 0.2881,\ A(0.4) = 0.1554,\ A(1.6) = 0.4452$)

6. Solve any TWO of the following: **12**

a) Attempt the following

i) Using Regula falsi method find the root of $xe^x - 3 = 0$ (Carry out 2 iterations)

ii) Solve the following system of equations by Gauss elimination method.

$x + 2y + 3z = 14,\ 3x + y + 2z = 11,\ 2x + 3y + z = 11$

b) Solve the following equation by Jaccobi's method. (two iterations)

$10x + y + 2z = 13,\ 3x + 10y + z = 14,\ 2x + 3y + 10z = 15.$

c) Using Newton Raphson method find the approximate root of the equation $x^4 - x - 9 = 0$ performing up to four iterations.