Important Instructions to examiners:
1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate’s answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate’s understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q.1. Attempt any FIVE of the following: 10 Marks

<table>
<thead>
<tr>
<th>Sr.No.</th>
<th>Parameter</th>
<th>Heat</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Definition</td>
<td>Form of energy that is transferred between system and surrounding or two systems due to temperature difference</td>
<td>The amount of energy transferred by a force acting through a distance</td>
</tr>
<tr>
<td>2.</td>
<td>Function</td>
<td>Heat is a function of the state</td>
<td>Heat is a function of the Path</td>
</tr>
<tr>
<td>3.</td>
<td>Energy Interaction</td>
<td>Due to Temperature Difference</td>
<td>Other than Temperature difference</td>
</tr>
</tbody>
</table>

b) State Clausius statement of second law of thermodynamics.

Sol. **Clausius statement**: It state that “It is impossible for a self-acting machine working in a cyclic Process without any external force, to transfer heat from a body at a lower temperature to a body at a higher temperature. Thus external mechanical work expenditure is necessary to transfer heat from a body at a low temperature to a body at high temperature.

![Diagram](image-url)
c) Define dryness fraction and degree of superheat.

**Sol.**

**Dryness Fraction:**
It is defined as a fraction of dry steam that is present in a liquid vapour is called dryness fraction.

**Or**

Dryness fraction is the ratio of the Mass of actual dry steam to the Mass of wet steam.

\[ X = \frac{M_s}{M_s + M_w} \]

Where \( X \) – Dryness fraction
\( M_s \) – mass of vapour (dry steam) contain in steam
\( M_w \) = mass of water in suspension in steam

**Degree of Superheat:**
The difference between the temperature of superheated steam and saturated steam \( (T_{\text{sup}} - T_{\text{sat}}) \) is known as degree of superheat.

Any one definition

1 mark

1 mark

d) Define Mach number and critical pressure.

**Sol.**

1. **Mach Number:** In fluid dynamics, the Mach number (M or Ma) is a dimensionless quantity representing the ratio of flow velocity past a boundary to the local speed of sound.

\[ M = \frac{c}{a} \]

\( M \) is the Mach number, \( c \) is the local flow velocity, \( a \) is the speed of sound in the medium

2. **Critical Pressure:** The Pressure for which the maximum discharge through nozzle occurs is called as critical pressure. It is denoted as \( P_c \)

1 mark

1 mark

e) Explain bleeding of steam.

**Sol.**

It is process of draining steam from turbine at certain point during its expansion and using these steams for heating the feed water supplied to boiler is known as bleed and the process is known as bleeding of steam.

1 mark

1 mark

f) State Dalton's law of partial pressure.

**Sol.**

This law states that “The total pressure exerted by a mixture of air and water vapour on the walls of container is the sum of partial pressure exerted by air separated and that exerted by vapour separately at common temperature of the condenser”.

\[ P = P_a + P_s \]

Where

\( P_a \) = partial pressure exhausted by air
\( P_s \) = partial pressure exhausted by vapour

\( P \) = total pressure of mixture at temperature.
g) **Define Fourier's law.**

**Sol.** The law states that for homogeneous material the rate of heat transfer in steady state in any direction is directly proportional to temperature gradient in that direction.

\[
\frac{Q}{A} \propto \frac{dt}{dx}
\]

\[
\frac{Q}{A} = -k \frac{dt}{dx}
\]

Where, \( \frac{Q}{A} \) is rate of heat transfer

\( \frac{dt}{dx} \) is temperature gradient

\( k \) conductivity of medium

---

Q.2. **Attempt any THREE of the following:**

---

**a) State extensive property and Intensive property with two examples each.**

**Sol.**

**Extensive property:** An extensive property of a system is one whose value depend upon the mass of the system.

e.g. volume, energy, enthalpy, entropy, internal energy.

**Intensive property:** An intensive property of a system is one whose value does not depend upon the mass of the system.

e.g. Density, Temperature, Pressure

---

**b) Define isentropic process and plot it on, P-V and T-S diagram.**

**Sol.**

**Isentropic Process:** The process in which working substance neither receives nor rejects heat to its surrounding during expansion or compression is called as Isentropic process, it is also known as adiabatic process.

Adiabatic process reversible when it is frictionless and the process is irreversible when it involves friction.

Process is denoted by equation \( PV^\gamma = C \)

![PV Diagram](image1)

![TS Diagram](image2)

---

01 mark

01 mark

1 mark

1 mark

1 mark

1 mark

2 marks

2 marks

(1 Mark for each Dig.)
c) Define:
(i) Sensible heat
(ii) Latent heat

Sol. i) Sensible Heat:
The heat in which change in temperature of a substance can be observed but phase remains unchanged that heat is known as Sensible heat.

This heat can be sensed by ordinary thermometer, It is given by the equation

Sensible heat = \( m \cdot Cp \cdot (T_2 - T_1) \)

\( m \) is mass
\( Cp \) is Specific heat at constant pressure
\( T_1 \) is Initial Temperature
\( T_2 \) is Final Temperature

ii) Latent Heat:
It is define as amount of heat required for the change of phase of 1 kg of water at saturated temperature to dry saturated steam at constant pressure .

It is denoted by \( L \) , Its value can be directly obtained from steam table

Heat at which solid changes phase to liquid is known as Latent heat of fusion
Heat at which Liquid Changes Phase to vapour is known as Latent heat of vaporization

<table>
<thead>
<tr>
<th>Sol. Sr. No.</th>
<th>Parameter</th>
<th>Water tube boiler</th>
<th>Fire tube boiler</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Medium in tube</td>
<td>Water is circulated in tube and hot gases passed over the tube</td>
<td>Hot gases are circulated through the tube and water flows over tube.</td>
</tr>
<tr>
<td>2.</td>
<td>Steam Formation Rate</td>
<td>Steam formation rate is high</td>
<td>Steam formation rate is low</td>
</tr>
<tr>
<td>3.</td>
<td>Steam Pressure</td>
<td>It can generate steam at higher pressure more than 25 bar</td>
<td>Generate steam at lower pressure up to 25 bar</td>
</tr>
<tr>
<td>4.</td>
<td>Operating cost</td>
<td>Operating cost high</td>
<td>Operating cost low</td>
</tr>
<tr>
<td>5.</td>
<td>Overall efficiency</td>
<td>Overall efficiency high</td>
<td>Overall efficiency low</td>
</tr>
<tr>
<td>6.</td>
<td>Cleaning and Inspection</td>
<td>Cleaning and Inspection is easy</td>
<td>Cleaning and Inspection is difficult</td>
</tr>
<tr>
<td>7.</td>
<td>Application</td>
<td>High power Generation</td>
<td>Low to medium power generation</td>
</tr>
<tr>
<td>8.</td>
<td>Example</td>
<td>Babcock &amp; Wilcox boiler, Loeffler Boiler</td>
<td>Cochran Boiler, Lancashire boiler</td>
</tr>
</tbody>
</table>

Q.3. Attempt any THREE of the following: 12 Marks
### a) State the term governing of turbine and explain nozzle control governing.

<table>
<thead>
<tr>
<th>Sol.</th>
<th>Governing of turbine:</th>
<th>01 mark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The function of governor is to regulate the supply of steam to the turbine so that the speed of rotation shall remain constant at all loads.</td>
<td></td>
</tr>
</tbody>
</table>

**Nozzle control governing:**

![Figure: Nozzle control governing](image)

- The arrangement of nozzle control governing is shown in figure.
- The poppet type valve uncover as many steam passages as necessary to meet the load, each passage serving a group of nozzle.
- The control governor has the advantage of using steam at full boiler pressure.
- The nozzles are divided into three groups $N_1$, $N_2$ and $N_3$ and the control valves $V_1$, $V_2$ and $V_3$ controls the amount of steam supply to each nozzle group respectively.
- The number of nozzle group may vary from three to five or more. Various arrangements of group nozzles and valves can be employed. Two arrangements are shown in figure (b) & (c).
- Under full load condition all the regulating valve are opened. When the load on the turbine is reduced the supply of steam to a group nozzle is shut off.

### b) Explain principle of working of Impulse steam turbine with neat sketch.

02 marks for Figure

01 mark explanation
Sol.

**Construction:**
Impulse turbine is simpler, less expensive and does not need to be pressure proof. It can operate with any pressure stream but is considerably less efficient. Impulse turbine consist of one fixed set of nozzle mounted on a stationary diaphragm that orient the steam flow into high speed jets, which is followed by one set of moving blade ring as shown in Fig. for a single stage impulse turbine.

| c) | A gas occupying 0.26 m$^3$ at 300°C and 0.4 MPa pressure expands till volume becomes 0.441 m$^3$ and pressure 0.26 MPa. Calculate the change in internal energy per kg of gas. $C_p = 1$ kJ/kg K, $C_v = 0.71$ kJ/kg K. |

Figure: Impulse Turbine

02 marks for figure
02 marks for explanation
Sol.

\[ \text{Q. 3. c.} \]

\[ \text{Soln: -} \]

Given data:

\[ V_1 = 0.26 \text{ m}^3, \quad T_1 = 800^\circ C = 300 + 273 = 573 \text{ K} \]

\[ P_1 = 0.4 \text{ MPa} \]

\[ V_2 = 0.441 \text{ m}^3 \]

\[ P_2 = 0.26 \text{ MPa} \]

\[ a_p = 1.4 \text{ kJ/kg}, \quad c_v = 0.71 \text{ kJ/kg.K} \]

Assume Temperature \( T_3 \) constant.

So,

The change in internal energy \( \Delta U = 0 \)

Assume Adiabatic process.

\[ P, V, T, S \text{ diagram of Adiabatic process} \]

\[ Y = \left( \frac{c_p}{c_v} \right) = 1.4 \text{ an Isentropic \( Y \) index.} \]

The change in internal energy \( \Delta U = m(c_v T_2 - T_1) \)

\[ T_2 = T_1 \times \left( \frac{V_1}{V_2} \right)^{\frac{c_p}{c_v} - 1} = 300 \times \left( \frac{0.26}{0.441} \right)^{1.4 - 1} \]

\[ T_2 = 242.836^\circ C \]

The change in internal energy

\[ \Delta U = 1 \times 0.71 \times (242.836 - 300) = -40.586 \text{ kJ/kg} \]

\[ \text{The negative sign indicates that internal energy is decreasing.} \]

02 marks

02 marks

(d) Determine the amount of heat supplied to 2 kg of water at 25°C to convert it into steam at 5 bar and 0.9 dry.
Q.4. Attempt any THREE of the following: 12 Marks

a) Differentiate between natural draught and forced draught cooling tower.

<table>
<thead>
<tr>
<th>Sol.</th>
<th>Sr. No.</th>
<th>Natural draught</th>
<th>Forced draught</th>
<th>01 mark for each differentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The air flows naturally without fan through tower and provides required cooling</td>
<td>The air circulation through the tower depends on wind velocity.</td>
<td>Fan is located at the top of the tower and enters the side of the tower.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>The air circulation through the tower depends on wind velocity.</td>
<td>The air circulation through the tower depends on fan speed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>The cooling Rate and efficiency of tower is less.</td>
<td>The cooling Rate and efficiency of tower is high.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>It requires large space for same capacity.</td>
<td>It requires less space for same capacity.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>No power requires due to absence of fan.</td>
<td>Fan requires more power as it handles hot air.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>The temp. of water coming out from</td>
<td>The temp. of water coming out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
b) A gas has a volume of 0.14 m³, pressure 1.6 bar and a temperature 110°C. If the gas is compressed at constant pressure until its volume becomes 0.112 m². Determine:
   i. Work done in compression of gas
   ii. Heat given out by gas

Sol.

\[ \Phi \text{ 4.b.} \]

**Given data:**

\[
V_1 = 0.14 \text{ m}^3 \\
P_1 = P_2 = 1.6 \text{ bar} = 1.6 \times 10^5 \text{ N/m}^2 \\
T_1 = 110 + 273 = 383 \text{ K} \\
V_2 = 0.112 \text{ m}^3 \\
\text{Assume } C_p = 1 \text{ kJ/kgK} \\
\]

**Figure:** P-V & T-s diagram of isobaric process

\[
P_1 V_1 = P_2 V_2 \\
\frac{T_1}{T_2} = \frac{V_2}{V_1} \\
\rightarrow T_2 = \frac{V_2 T_1}{V_1} = \frac{0.112 \times 383}{0.14} = 306.4 \text{ K} \\
\]

\[
1) \text{ Work done in compression of gas} \\
\begin{align*}
\text{dW} &= P \text{ CV}_2 - V_1 \\
&= 1.6 \times 10^5 \times (0.112 - 0.14) \\
&= -4.980 \text{ J} \\
&= -4.98 \text{ kJ}
\end{align*}
\]

\[
2) \text{ Heat given out by gas} \\
\begin{align*}
\text{dQ} &= dU + df \text{W} = mc_p(T_2 - T_1) \\
&= 1 \times 1 \times (306.4 - 383) \\
&= -76.6 \text{ kJ}
\end{align*}
\]

c) A certain gas has \( C_p = 1.968 \text{ kJ/kg K} \), \( C_v = 1.507 \text{ kJ/kgK} \). Find the molecular weight and the gas constant. A constant volume chamber of
0.3 m³ capacity contain 2 kg of this gas at 5°C. Heat is transferred to the gas until the temperature is 100°C. Find the work done and change in internal energy.

Sol.

\[
\begin{align*}
\text{Given data:} & \\
C_p &= 1.988 \text{ kJ/kg}^o \text{k} \\
C_v &= 1.507 \text{ kJ/kg}^o \text{k} \\
\text{Constant volume, } V &= 0.3 \text{ m}^3 \\
M &= 2 \text{ kg} \\
T_1 &= 273 + 273 = 273 \text{ K} \\
T_2 &= 100 + 273 = 373 \text{ K} \\
\frac{V}{M} &= 8.315 \\
M &= 2.3143 \\
\mu &= 0.441 \\
\rho &= 18 \text{ kg} \\
\end{align*}
\]

\[
\begin{align*}
3) & \quad \text{Heat transfer } = Q = mC_v(T_2 - T_1) \\
& = 2.3143 \times (373 - 273) = 2.811 \text{ kJ} \\
\text{d) Work done } = W = \int P \, dv \\
& = -\frac{\Delta V}{\Delta P} = 0 \\
\text{So, Work done } = W = 0 \\
\end{align*}
\]

01 mark each answer

---

**Define:**

i. Transmissivity
ii. Black body
iii. Grey body
iv. Reflectivity

**Sol.**

**Transmissivity:**
It is the fraction of energy which is transmitted through the body.

Or
The ratio of amount of energy transmitted to the amount of energy incident on a body.

**Black body:** A black body is an object that absorbs all the radiant energy reaching its surface from all the direction with all the wavelengths. Gray body:

**Grey Body:** A gray body is defined as a body whose absorptivity of a surface does not vary with variation in temperature and wavelength of the incident radiation. It absorbs a
definite percentage of incident energy irrespective of wavelength. Its absorptivity lies between 0 to 1.

**Reflectivity:**
It is defined as the ratio of amount of energy reflected to the amount of energy incident on a body.

e) **Draw a neat sketch of surface condenser and label it.**

**Sol.**

![Surface Condenser Sketch](image)

**Figure: Surface Condenser**

<table>
<thead>
<tr>
<th>Q.5. Attempt any TWO of the following:</th>
<th>12 Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) List out any six losses in steam turbine.</td>
<td></td>
</tr>
<tr>
<td><strong>Sol.</strong> <strong>Losses in steam turbine</strong></td>
<td></td>
</tr>
<tr>
<td>1. Residual velocity loss</td>
<td></td>
</tr>
<tr>
<td>2. Loss due to friction</td>
<td></td>
</tr>
<tr>
<td>3. Leakage loss</td>
<td></td>
</tr>
<tr>
<td>4. Loss due mechanical friction</td>
<td></td>
</tr>
<tr>
<td>5. Radiation loss</td>
<td></td>
</tr>
<tr>
<td>6. Loss due to moisture</td>
<td></td>
</tr>
<tr>
<td>7. Carry over losses</td>
<td></td>
</tr>
<tr>
<td>b) A steel pipe of inner and outer diameter 6 cm and 8 cm respectively has inside temperature 140°C and outside temperature 50°C. The thermal conductivity of steel is 24 W/mK. Calculate the rate of heat transfer through the pipe if length of pipe is 1.5 m.</td>
<td></td>
</tr>
<tr>
<td><strong>Sol.</strong> <strong>Given Data:</strong> Length of pipe = L = 1.5 m, Inner diameter, d1 = 6 cm, Outer diameter, d2 = 8 cm, Inside temp T1 = 140°C, Outside temp T2 = 40°C, Ksteel = 24 W/MK</td>
<td></td>
</tr>
<tr>
<td>**Q = ( \frac{T1 - T2}{\frac{1}{2\pi LR} \log\frac{r2}{r1}} )</td>
<td></td>
</tr>
</tbody>
</table>
\[ Q = \frac{140 - 40}{\left(\frac{2\pi 	imes 1.5 	imes 24}{100}\right) \log \frac{3}{1.272 	imes 10^{-3}}} \]

\[ Q = 78616.35 \text{ Watts or 78.62 KW} \]

c) List any SIX methods of energy conservation in boilers.

**Sol.** List any six methods of energy conservation in boilers

**Following methods can conserve energy in boilers**

1) Reduction radiation and convention losses
2) Waste heat recovery for heating to the feed water.
3) Continues monitoring of flue gases losses and other losses
4) Using standard efficient fuel firing equipments, burners, mechanical stockers.
5) Scheduling boiler operation to avoid fluctuation in boiler load
6) Installation of variable speed drives.
7) Optimise boiler stem pressure and temperature
8) Periodic energy audit.

Periodic preventive maintenance of all components.

Q.6. Attempt any TWO of the following:

12 Marks

a) **Explain the necessity of compounding in steam turbine and draw a neat sketch of pressure velocity compounding.**

**Sol.** **Necessity of compounding in steam turbine:** Compounding of steam turbines is necessary 1) To reduce speed of rotor blades to practical limits.
2) To reduce centrifugal force and hence to prevent failure of blades.
3) To reduce velocity of steam leaving blades.

If entire pressure drop from boiler pressure to condenser pressure is carried out in a single stage of nozzle then the velocity of steam entering the turbine blades will be very high. The turbine speed has to be also very high as it is directly proportional to steam velocity. Such high rpm of turbine rotor are not useful for practical purposes & there is a danger of structural failure of blades due to excessive centrifugal stresses. Hence compounding is carried out.

**Neat sketch of pressure velocity compounding.**

01 mark each

03 marks

03 marks for figure
Figure: Pressure and Velocity compounding

b) Explain the application of second law of thermodynamics to refrigerator. State any three functions of steam condenser.

Sol. According to Clausius’s statement “heat cannot flow itself from cold body to a hot body without help of external agency”

\[
\text{REFRIGERATOR is shown in the figure with cold body (T2) and hot body (atmosphere) (T1). A refrigerator is a device which maintains the temperature of cold body below the surrounding temp. The amount of heat taken from cold body which is to be cooled is } Q_2. \text{ For doing this work, external energy is required to the refrigerator. So heat rejected to atmosphere } Q_1 = Q_2 + W. \\
\text{As per the statement of Statement of Second Law of Thermodynamics, it is observed that refrigerator operates between the two different temperatures in a cyclic manner. It also extracts the heat from cold body only (storage space) and does the equivalent amount of work as shown.}
\]
In a full cycle of a refrigerator, three things happen:
1. Heat is absorbed from cold body, the heat can be called Q2.
2. Some of the energy from that input heat is used to perform work (W).
3. The rest of the heat is rejected to hot body (Q1).

An performance of the refrigerator can be calculated as: Efficiency = Q1 / work W
So it is cleared that the external energy is required to absorb heat from cold body and to reject it to hot body.

**Function of condenser:**
1) **To maintain a very low back pressure** so as to obtain the maximum possible energy from steam and thus secure a high efficiency.
2) **To condense the steam and reuse it** to supply as pure feed water to the hot well from where it is pumped back to the boiler.
3) **To remove of air and non-condensable gases**

**c) Derive characteristic gas equation using Boyle’s and Charle’s law.**

**Sol.**

Characteristic gas equation using Boyle’s & Charle’s law:

Let us consider a unit mass of an ideal gas to change its state in following two processes as shown in fig.

Here, process 1-2’ is at constant pressure
Process 2’-2 is at constant temperature
Now, applying Charle’s law for process 1-2’
We get

\[
\frac{V_1}{T_1} = \frac{V_2'}{T_2} \quad \text{as} (T_2' = T_2) \quad \ldots \ldots \ldots (I)
\]

Now, applying Boyle’s law for process 2’-2,
\[ P_2' V_2 = P_2 V_2 \quad (T = C) \]
\[ P_1 V_2' = P_2 V_2 \quad (\text{As } P_2' = P_1) \]
\[ V_2' = \frac{P_2 V_2}{P_1} \quad \ldots \ldots \ldots (II) \]

Substituting eq (II) IN eq (I), We get
\[ \frac{V_1}{T_1} = \frac{P_2 V_2}{P_1 T_2} \]
\[
\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}
\]

i.e.
\[
\frac{PV}{T} = \text{Constant} = R \quad \ldots \ldots \quad (III)
\]

Consider m kg of gas, multiply eq (III) by m
\[
\frac{mPV}{T} = R \times m
\]

\[
\frac{PV}{T} = R \times m \quad \text{here} \quad V = V_m = \text{total volume}
\]

Therefore
\[
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