Model Answers

Winter – 2018 Examinations

Subject & Code: Electrical Circuits (22324)

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Model Answers
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Subject & Code: Electrical Circuits (22324)

1 Attempt any FIVE of the following: 10

1 a) Define active power and reactive power for RLC series circuit.

Ans:

Active Power (P):

Active power (P) is given by the product of voltage, current and the cosine of the phase angle between voltage and current. 1 mark

Unit: watt (W) or kilo-watt (kW) or Mega-watt (MW).

$P = VI\cos\phi = I^2R$ watt

Reactive Power (Q):

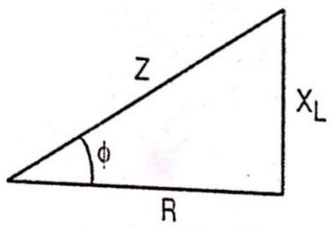
Reactive power (Q) is given by the product of voltage, current and the sine of the phase angle between voltage and current. 1 mark

Unit: volt-ampere-reactive (VAr), or kilo-volt-ampere-reactive (kVAr) or Mega-volt-ampere-reactive (MVAr)

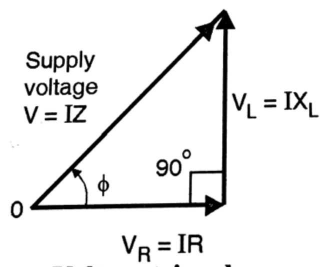
$Q = VI\sin\phi = I^2X$ volt-amp-reactive.

1 b) Draw impedance triangle and voltage triangle for RL series circuit.

Ans:



Impedance Triangle



Voltage triangle

1 mark for each triangle = 2 marks

1 c) Define susceptance and admittance for parallel circuit.

Ans:

Admittance (Y):

Admittance is defined as the ability of the circuit to carry (admit) alternating current through it. It is the reciprocal of impedance Z. i.e Y = 1/Z. 1 mark

For parallel circuit consisting two branches having impedances Z_1 and Z_2 in parallel, the equivalent impedance of parallel combination is given by, (Equations are optional)

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y = Y_1 + Y_2$$

where, Y is the equivalent admittance of the parallel circuit

Y_1 and Y_2 are the admittances of the two branches respectively.

If the equivalent impedance is expressed as $Z = R + jX$, then the admittance is obtained as,

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{(R + jX)(R - jX)} = \frac{R - jX}{R^2 + X^2}$$

$$\therefore Y = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = G - jB$$

Susceptance (B):

Susceptance is defined as the imaginary part of the admittance.
It is expressed as, 1 mark

$$B = \frac{X}{R^2+X^2}$$

In DC circuit, the reactance is absent, hence X = 0 and susceptance equals to zero.

1 d) Define quality factor for parallel resonance and write its mathematical expression.

Ans:

Quality Factor of Parallel AC Circuit at resonance:

The quality factor or Q-factor of parallel circuit is defined as the ratio of the current circulating between two branches of the circuit to the current taken by the parallel circuit from the source. 1 mark

It is the current magnification in parallel circuit.

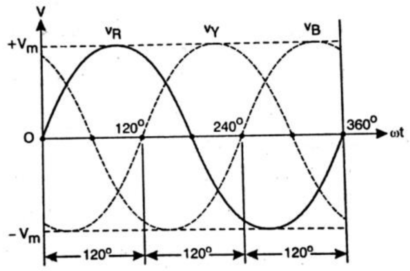
Formula:

Quality factor (Q-factor) = Current magnification $= \frac{1}{R}\sqrt{\frac{L}{C}}$ 1 mark

Where, R is the resistance of an inductor in Ω,
L is the inductance of an inductor in henry,
C is capacitance of capacitor in farad,

1 e) Draw sinusoidal waveform of 3 phase emf and indicate the phase sequence.

Ans –



1½ marks for waveform

½ mark for phase sequence

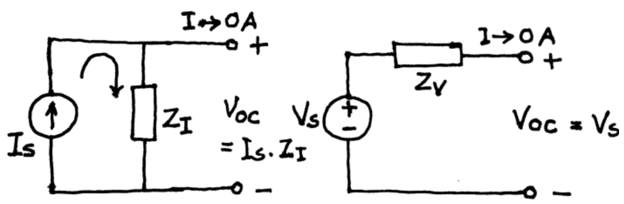
Phase sequence is R-Y-B.

1 f) Write the procedure of converting a given current source into voltage source.

Ans:

Conversion of current source into equivalent voltage source:

Let I_S be the practical current source magnitude and
Z_I be the internal parallel impedance.
V_S be the equivalent practical voltage source magnitude and
Z_V be the internal series impedance of the voltage source.

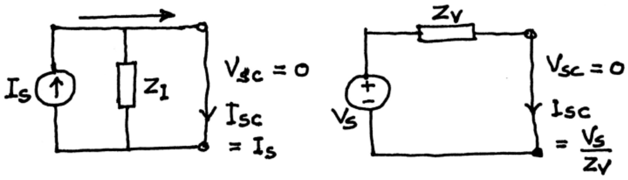


1 mark for diagram
1 mark for description

The open circuit terminal voltage of current source is $V_{OC} = I_S \times Z_I$

The open circuit terminal voltage of voltage source is $V_{OC} = V_S$
Therefore, we get $V_S = I_S \times Z_I$ ………………………….(1)

OR



The short circuit output current of current source is $I_{SC} = I_S$
The short circuit output current of voltage source is $I_{SC} = V_S / Z_V$
Therefore, we get $I_S = V_S / Z_V$ ………………………….(2)
On comparing eq. (1) and (2), it is clear that $Z_I = Z_V = Z$ ……….. …..(3)
Thus the internal impedance of both the sources is same, and the magnitudes of the source voltage and current are related by Ohm's law, $V_S = I_S \times Z_I$

1 g) State superposition theorem applied to the d.c. circuits.
Ans:
Superposition Theorem applied to D.C. circuits:
Superposition theorem states that in any linear, bilateral, multisource network, the response (voltage across any element or current through any element) of any branch is equal to the algebraic sum of the responses produced in it with each source acting alone, while the other sources are replaced by their internal resistances. 2 marks

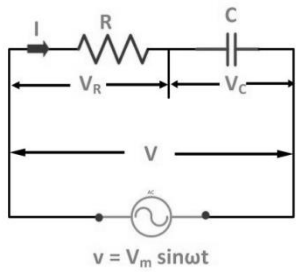
OR

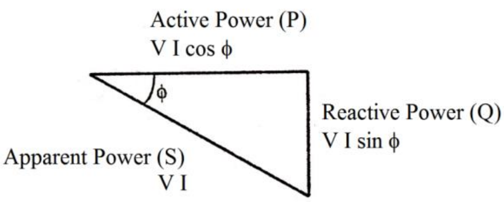
Any other equivalent valid statement

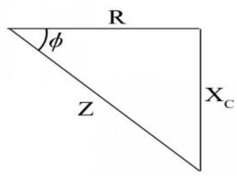
2 Attempt any THREE of the following: 12

2 a) Draw a circuit diagram of R.C. series circuit. Draw impedance triangle and power triangle for same circuit.
Ans –

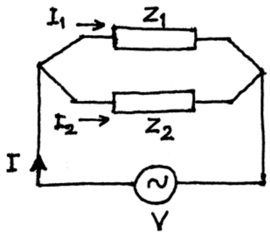
1 mark for circuit
½ mark for each of power & voltage triangle







2 b) Two circuits the impedance of which are given by $Z_1 = 6 + j8$ ohm and $Z_2 = 8 – j6$ ohm are connected in parallel. If the applied voltage to the combination is 100V, Find:
(i) Current and power factor at each branch.
(ii) Overall current and power factor of the combination.
(iii) Power consumed by each impedance. Draw phasor diagram.
Ans:



Data given:

$Z_1 = 6 + j8 = 10\angle 53.13° \ \Omega$

$Z_2 = 8 - j6 = 10\angle -36.87° \ \Omega$

$V = 100 + j0 = 100\angle 0°$ volt — 1 mark

(i) Current and power factor at each branch:

Current of branch 1: $I_1 = V/Z_1 = \frac{100\angle 0°}{10\angle 53.13°} = \mathbf{10\angle -53.13°\ A}$

Power factor of branch 1: cos(53.13°) = **0.6 lagging**

Current of branch 2: $I_2 = V/Z_2 = \frac{100\angle 0°}{10\angle -36.87°} = \mathbf{10\angle 36.87°\ A}$

Power factor of branch 2: cos(36.87°) = **0.8 leading**

1 mark

(ii) Overall current and power factor of the combination:

Over all current $I = I_1 + I_2 = 10\angle -53.13° + 10\angle 36.87°$

$= 6 - j8 + 8 + j6 = 14 - j2 = \mathbf{14.14\angle -8.13°\ A}$

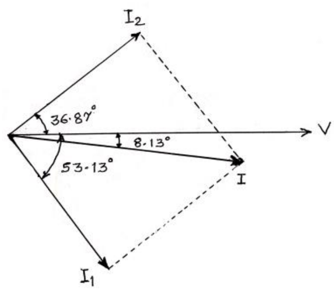
Overall power factor: cos(8.13°) = **0.98 lagging** — 1 mark

(iii) Power consumed by each impedance:

Power consumed by Z_1 : $V.I_1.\cos\phi_1$ = (100)(10)cos(53.13°) = **600W** **OR**
$I_1^2R_1 = (10)^2(6) = 600W$

Power consumed by Z_2 : $V.I_2.\cos\phi_2$ = (100)(10)cos(36.87°) = **800W** **OR**
$I_2^2R_2 = (10)^2(8) = 800W$

(iv) Phasor diagram: — 1 mark



2 c) State any four advantages of polyphase circuits over single phase circuit.

Ans:

Advantages of polyphase (3-phase) circuits over Single-phase circuits:

i) Three-phase transmission is more economical than single-phase transmission. It requires less copper material.

ii) Parallel operation of 3-phase alternators is easier than that of single-phase alternators.

iii) Single-phase loads can be connected along with 3-ph loads in a 3-ph system.

1 mark for each of any four = 4 marks

iv) Instead of pulsating power of single-phase supply, constant power is obtained in 3-phase system.
v) Three-phase induction motors are self-starting. They have high efficiency, better power factor and uniform torque.
vi) The power rating of 3-phase machine is higher than that of 1-phase machine of the same size.
vii) The size of 3-phase machine is smaller than that of 1-phase machine of the same power rating.
viii) For same power rating, three-phase motors are cheaper than the single-phase motors.

2 d) Using mesh analysis, find loop currents I_1 and I_2 in the circuit, as shown in fig. no. 1

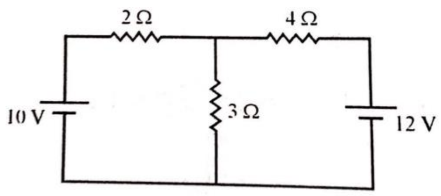
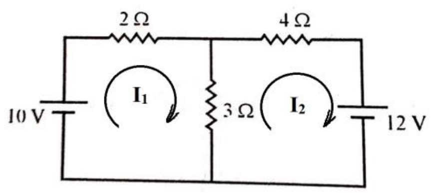


Fig. 1

Ans:
Mesh Analysis:
i) There are two meshes in the network.



ii) Mesh currents I_1 and I_2 are marked clockwise as shown.
iii) By tracing mesh 1 clockwise, KVL equation is,
$10 - 2I_1 - 3(I_1 - I_2) = 0$
$\therefore 5I_1 - 3I_2 = 10 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$ 1 mark
By tracing mesh 2 clockwise, KVL equation is,
$-4I_2 - 12 - 3(I_2 - I_1) = 0$
$\therefore 3I_1 - 7I_2 = 12 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$ 1 mark
iv) Expressing eq.(1) and (2) in matrix form,

$$\begin{bmatrix} 5 & -3 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 5 & -3 \\ 3 & -7 \end{vmatrix} = -35 - (-9) = -26$$

By Cramer's rule,

$$I_1 = \frac{\begin{vmatrix} 10 & -3 \\ 12 & -7 \end{vmatrix}}{\Delta} = \frac{(-70) - (-36)}{-26} = \frac{-34}{-26} = \mathbf{1.307\ A}$$ 1 mark

$$I_2 = \frac{\begin{vmatrix} 5 & 10 \\ 3 & 12 \end{vmatrix}}{\Delta} = \frac{(60) - (30)}{-26} = \frac{30}{-26} = \mathbf{-1.154\ A}$$ 1 mark

3 Attempt any THREE of the following: 12

3 a) Derive the expression for resonance frequency for a RLC series circuit.

Ans:

Resonant Frequency of Series RLC Circuit:

In RLC series circuit the resonance occurs when the inductive reactance (X_L) becomes equal to the capacitive reactance (X_C). *1 mark for equations of reactances*

Inductive reactance is given by $X_L = 2\pi fL$

Capacitive reactance is given by $X_C = \frac{1}{2\pi fC}$

The inductive reactance (X_L) becomes equal to capacitive reactance (X_C) only at one particular frequency, which is known as resonant frequency and it is denoted by f_r. *1 mark for resonance condition*

Hence at resonance,

$$X_L - X_C = 0$$

$$X_L = X_C$$

$$2\pi\, f_r L = \frac{1}{2\pi f_r C}$$ 1 mark

Rearranging above equation,

We get,

$$(f_r)^2 = \frac{1}{4\pi^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\ \text{Hz} \qquad \text{OR} \qquad \omega_r = \frac{1}{\sqrt{LC}}\ \text{rad/sec}$$ 1 mark

3 b) Compare series resonance to parallel resonance on the basis of

(i) Resonant frequency (ii) Impedance (iii) Current (iv) Magnification

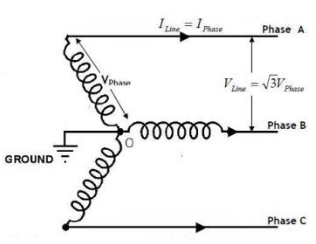
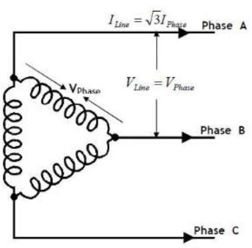
Ans:

Parameter	Series Resonant Circuit	Parallel Resonant Circuit
Resonant frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz	$f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz
Impedance	Minimum Z= R ohms	Maximum $Z = \frac{L}{CR}\ \Omega$
Current	Maximum $I = \frac{V}{R}\ \Omega$	Minimum $I = \frac{V}{\frac{L}{CR}}\ \Omega$
Magnification	Voltage magnification	Current magnification

1 mark for each point = 4 marks

3 c) Compare star and delta connection.(Any four points)

Ans:

Parameter	Star Connection	Delta Connection
Basic definition	One terminal of each of the three branches are connected together to form a common point. Such a connection is known as Star Connection	The three branches of the network are connected in such a way that it forms a closed loop. Such a connection is known as Delta Connection
Connection of terminals	The similar ends of the three coils are connected together to form a common point.	The end of each coil is connected to the starting point of the other coil that means the opposite terminals of the coils are connected together to form a closed loop.
Neutral point	Neutral or the star point exists in the star connection.	Neutral point does not exist in the delta connection.
Relation between line and phase current	Line current is equal to the Phase current.	Line current is equal to $\sqrt{3}$ times the Phase Current.
Relation between line and phase voltage	Line voltage is equal to $\sqrt{3}$ times the Phase Voltage	Line voltage is equal to the Phase voltage.
Diagram		

1 mark for each of any four points = 4 marks

3 d) By using nodal analysis, calculate the current in 110Ω resister and p.d. across 110Ω resistor as shown in fig. no. 2

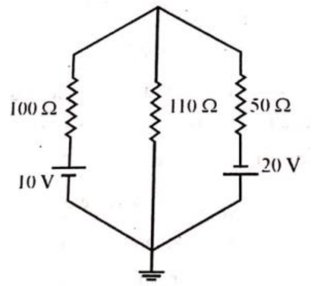
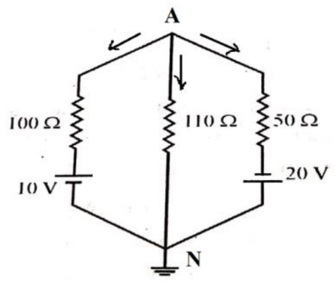


Fig. 2

Ans:



1 mark for diagram with currents

By applying KCL at node A, the node voltage equation can be written as:

$$\frac{V_A-10}{100}+\frac{V_A}{110}+\frac{V_A-(-20)}{50}=0$$ 1 mark

$$V_A\left(\frac{1}{100}+\frac{1}{110}+\frac{1}{50}\right)-\left(\frac{10}{100}-\frac{20}{50}\right)=0$$

$$V_A(0.0391)-(-0.3)=0$$

$$V_A=\frac{-0.3}{0.0391}=\mathbf{-7.67\ volt}$$

∴ P. D. across 110Ω resistor is $V_A = \mathbf{-7.67\ volt}$ 1 mark

(Terminal N is at higher potential than terminal A)

∴Current flowing through 110Ω is given by,

$$I=\frac{V_A}{110}=\frac{-7.67}{110}=-0.0697A$$ 1 mark

∴ I = 0.0697A flowing from terminal N to A

3 e) Convert following circuit as shown in fig. no. 3 into Thevenin's circuit across A & B.

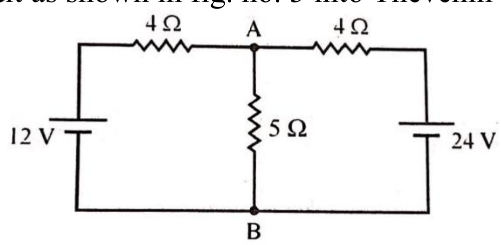
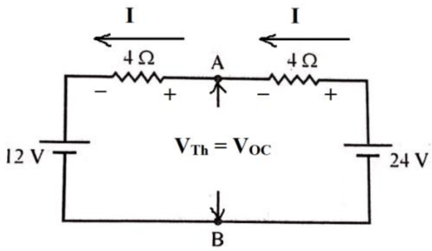


Fig. 3

Ans:

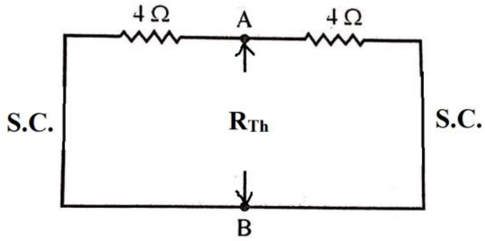
Determination of Thevenin's Equivalent Voltage Source (V_{Th}):



½ mark for figure

Thevenin's equivalent voltage source V_{Th} is the open circuit voltage across the load terminals A-B due to internal sources, as shown in the figure.
By tracing loop in anti-clockwise direction, the voltage equation can be written as:
24 – 4I – 4I – 12 = 0
Circuit current I = (24-12)/8 = 1.5 A
The Thevenin's equivalent voltage is given by,
$V_{Th} = V_{OC}$ = 24 – 4I = 24 – 6 = **18 volt** OR 1 mark
= 12 + 4I = 12 + 6 = **18 volt**

Determination of Thevenin's Equivalent Resistance (R_{Th}):



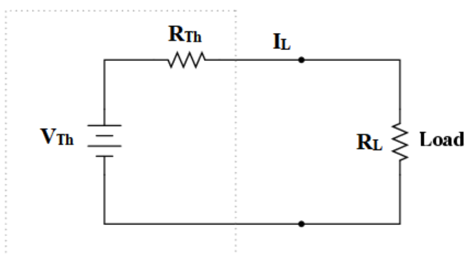
½ mark for figure

Thevenin's equivalent resistance is the resistance seen between the open-circuited load terminals while looking back into the network, with internal independent voltage sources replaced by short-circuit and independent current sources replaced by open-circuit, as shown in the following figure.

$$\mathbf{R_{Th}} = (4||4) = \frac{4 \times 4}{4+4} = \mathbf{2\ \Omega}$$ 1 mark

Thevenin's Equivalent Circuit:

Thevenin Equivalent Circuit



1 mark

$\mathbf{V_{Th} = 18V}$, $\mathbf{R_{Th} = 2\ \Omega}$, $\mathbf{R_L = 5\ \Omega}$

4 **Attempt any THREE of the following.** **12**

4 a) A resistance of 100Ω, an inductance of 0.2 H and capacitance of 150 μF are connected in series across 230V, 50 Hz ac supply. Calculate the current drawn by the circuit, power factor of the circuit, its nature and power consumed by the circuit.

Ans:

Given: R = 100 Ω, L = 0.2H, C = 150 μF = 150×10^{-6} F, V = 230V, f = 50 Hz

$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.2 = 62.83\ \Omega$

$X_C = 1/(2\pi f C) = 1/(2\times\pi\times50\times150\times10^{-6}) = 21.22\ \Omega$

Impedance $= Z = \sqrt{[R^2+(X_L-X_C)^2]} = \sqrt{\{100^2 + (62.83-21.22)^2\}} = \mathbf{108.31\ \Omega}$ **OR**

$= 100+j(62.83 - 21.22) = (100 + j41.61) = \mathbf{108.31\angle 22.59°\Omega}$ 1 mark

(1) Total current $= I = V/Z = 230\angle 0°/ 108.31\angle 22.59° = \mathbf{2.123\angle - 22.59°\ A}$ 1 mark

(2) Power factor = cos Ø = R/Z = 100/ 108.31 = **0.923 lagging** **OR**

= cos(22.59°) = **0.923 lagging** 1 mark

(3) Nature of power factor is **lagging**.

(4) $P = I^2 R = 2.123^2 \times 100 =$ **450.7 watt** **OR** 1 mark

P = V I cos Ø = 230 x 2.123 x 0.923 = **450.7 watt**

4 b) Define: (i) Admittance (ii) Susceptance
(iii) Conductance (iv) State the units of admittance and conductance

Ans:

(i) Admittance (Y):

Admittance is defined as the ability of the AC circuit to carry (admit) alternating current. It is also defined as reciprocal of impedance (Y). 1 mark

Admittance $(Y) = \frac{1}{Z}$ mho (℧)

(ii) Susceptance (B):

It is imaginary part of the admittance (Y). It is defined as the ability of the purely reactive circuit (purely capacitive or purely inductive) to admit alternating current. 1 mark

OR

It is ratio of reactance (X) to squared impedance (Z^2).

In general, Susceptance $(B) = \frac{X}{Z^2}$ siemen

(iii) Conductance(G):

It is defined as the real part of the admittance (Y). It is also defined as the ability of the purely resistive circuit to pass the alternating current. 1 mark

OR

It is the ratio of resistance (R) to squared impedance (Z^2)

Conductance$(G) = \frac{R}{Z^2}$ siemen

(iv) Units of admittance and conductance: 1 mark

Unit of Admittance (Y) = **mho**

Unit of Conductance (G) = **siemen**

4 c) Delta connected induction motor is supplied by 3 phase 400V, 50 Hz supply the line current is 43.03 amp and the total power from the supply is 24 kW. Find the resistance and reactance per phase of the motor.

Ans:

Data Given:

$V_L = 400V$, 3 ϕ (Delta connected) $I_L = 43.03$ A

P = 24 kW f = 50 Hz

In Delta connection $I_L = \sqrt{3}\ I_{Ph}$

$I_{ph} = \frac{I_L}{\sqrt{3}}$

$I_{ph} = \frac{43.03}{\sqrt{3}}$

$\mathbf{I_{ph} = 24.84\ A}$ ½ mark

In Delta connection $V_L = V_{ph}$

Hence,

$V_{ph} = V_L = 400\ V$

Total three-phase power supplied to motor is given by,

$P = 3\ V_{ph}\ I_{ph}\ \cos\phi$

$\cos\phi = \frac{P}{3x\, V_{ph} I_{ph}} = \frac{24\ x\ 10^3}{3x\ 400\ x\ 24.84}$ = **0.805 lagging** ½ mark

i) Impedance per phase Z_{ph}:

$Z_{ph} = \frac{V_{ph}}{I_{ph}}$

$Z_{ph} = \frac{400}{24.84}$

Impedance per phase $\mathbf{Z_{ph} = 16.10\Omega}$ 1 mark

ii) Resistance per Phase R_{ph}:

$\cos\phi = \frac{R_{ph}}{Z_{ph}}$ $R_{ph} = Z_{ph} \cos\phi$

R_{ph}= 16.10 x 0.805

$\mathbf{R_{ph} = 12.96\ \Omega}$ 1 mark

iii) Reactance per Phase X_{Lph}:

$X_{Lph} = \sqrt{(Z_{ph})^2 - (R_{ph})^2}$

$X_{Lph} = \sqrt{(16.10)^2 - (12.96)^2}$

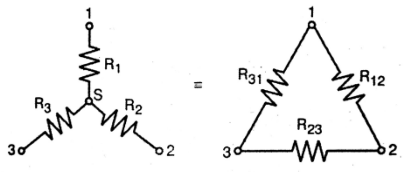
$\mathbf{X_{Lph} = 9.55\ \Omega}$ 1 mark

(Correct solution by ant other method may please be considered)

4 d) Derive the formulae for star to delta transformation.

Ans:

Star-delta Transformation:



(a) Star Circuit (b) Delta Circuit

If the star circuit and delta circuit are equivalent, then the resistance between any two terminals of the circuit must be same.

For star circuit, resistance between terminals 1 & 2, say $R_{1\text{-}2} = R_1 + R_2$

For delta circuit, resistance between terminals 1 & 2, $R_{1\text{-}2} = R_{12}||(R_{31} + R_{23})$

$$\therefore R_1 + R_2 = R_{12}||(R_{31}+R_{23}) = \frac{R_{12}(R_{31}+R_{23})}{R_{12} + (R_{31}+R_{23})} = \frac{R_{12}(R_{31}+R_{23})}{R_{12}+R_{23} + R_{31}}$$

$$\therefore R_1 + R_2 = \frac{R_{12}R_{31}+R_{12}R_{23}}{R_{12}+R_{23} + R_{31}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Similarly, the resistance between terminals 2 & 3 can be equated as,

$$\therefore R_2 + R_3 = \frac{R_{12}R_{23} + R_{23}R_{31}}{R_{12}+R_{23} + R_{31}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

And the resistance between terminals 3 & 1 can be equated as,

$$\therefore R_3 + R_1 = \frac{R_{23}R_{31} + R_{12}R_{31}}{R_{12}+R_{23} + R_{31}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Subtracting eq. (2) from eq.(1),

$$\therefore R_1 - R_3 = \frac{R_{12}R_{31} - R_{23}R_{31}}{R_{12}+R_{23} + R_{31}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Adding eq.(3) and eq.(4) and dividing both sides by 2,

$$\therefore R_1 = \left[\frac{R_{12}R_{31}}{R_{12}+R_{23} + R_{31}}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

1 mark for (eq.5, 6 & 7)

Similarly, we can obtain,

$$\therefore R_2 = \left[\frac{R_{12}R_{23}}{R_{12}+R_{23} + R_{31}}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\therefore R_3 = \left[\frac{R_{31}R_{23}}{R_{12}+R_{23} + R_{31}}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Multiplying each two of eq.(5), (6) and (7),

$$\therefore R_1R_2 = \left[\frac{(R_{12})^2R_{31}\,R_{23}}{(R_{12}+R_{23} + R_{31})^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

1 mark for (eq.8, 9 & 10)

$$\therefore R_2R_3 = \left[\frac{(R_{23})^2R_{31}\,R_{12}}{(R_{12}+R_{23} + R_{31})^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$\therefore R_3R_1 = \left[\frac{(R_{31})^2R_{12}\,R_{23}}{(R_{12}+R_{23} + R_{31})^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Adding the three equations (8), (9) and (10),

$$\therefore R_1R_2 + R_2R_3 + R_3R_1 = \frac{(R_{12})^2R_{31}\,R_{23}+(R_{23})^2R_{31}\,R_{12} + (R_{31})^2R_{12}\,R_{23}}{(R_{12}+R_{23} + R_{31})^2}$$

$$= \frac{R_{12}R_{31}\,R_{23}(R_{12}+R_{23} + R_{31})}{(R_{12}+R_{23} + R_{31})^2}$$

$$\therefore R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{31}\,R_{23}}{R_{12}+R_{23} + R_{31}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

1 mark for eq. 11

Dividing eq.(11) by eq.(6), (dividing by respective sides)

$$\therefore R_1 + R_3 + \frac{R_3R_1}{R_2} = R_{31}$$

$$\therefore R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Similarly, we can obtain,

$$\therefore R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

1 mark for (eq. 12, 13 & 14)

$$\therefore R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Thus using known star connected resistors R_1, R_2 and R_3, the unknown resistors R_{12}, R_{23} and R_{31} of equivalent delta connection can be determined.

Model Answers

Winter – 2018 Examinations

Subject & Code: Electrical Circuits (22324)

5 Attempt any TWO of the following: 12

5 a) A choke coil has a resistance of 4Ω and inductance of 0.07H is connected in parallel with another coil of resistance 10Ω and inductance 0.12H. The combination is connected to 230V, 50Hz supply. Determine total current and current through each branch.

Ans:

Data Given: $R_1 = 4\ \Omega$ $L_1 = 0.07H$ $R_2 = 10\ \Omega$ $L_2 = 0.12H$ $V = 230V$, $f = 50Hz$

$X_{L1} = 2\pi fL_1 = 2\pi(50)(0.07) = 21.99 \cong 22\ \Omega$

$X_{L2} = 2\pi fL_2 = 2\pi(50)(0.12) = 37.7\ \Omega$

½ mark for each of X_{L1} & X_{L2}

$Z_1 = R_1 + j\,X_{L1} = (4+j22) = 22.35\angle 79.7°\ \Omega$ 1 mark

$Z_2 = R_2 + j\,X_{L2} = (10+j37.7) = 39\angle 75.144°\ \Omega$ 1 mark

Branch 1 current is given by,

$I_1 = \frac{V}{Z_1} = \frac{230\angle 0°}{22.35\angle 79.7°} = \mathbf{10.3\angle\text{-}79.7°A = (1.84 - j10.13)\ A}$ 1 mark

Branch 2 current is given by,

$I_2 = \frac{V}{Z_2} = \frac{230\angle 0°}{39\angle 75.144°} = \mathbf{5.89\angle\text{-}75.144°A = (1.51 - j5.7)\ A}$ 1 mark

Total current is,

$I = I_1 + I_2 = (1.84 - j10.13) + (1.51 - j5.7)$

$\mathbf{I = (3.35 - j15.825)A = 16.17\angle\text{-}78.04\ \ A}$ 1 mark

5 b) Determine the current in 40Ω and 10Ω as shown in fig. no. 4 by node voltage analysis method.

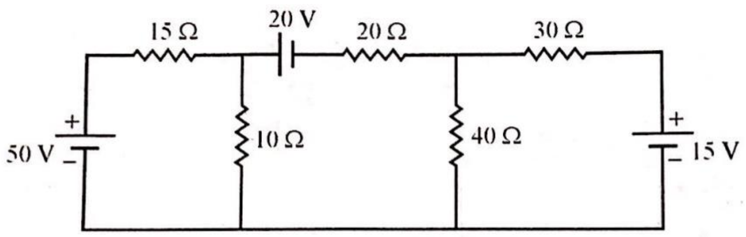
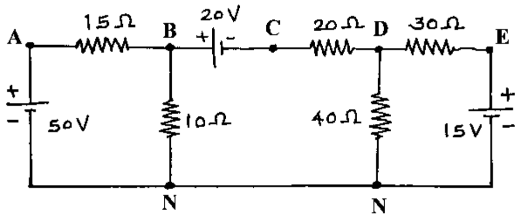


Fig. 4

Ans:

Node Voltage Analysis Method:

Step I: Mark the nodes and reference node.



1 mark for node identification

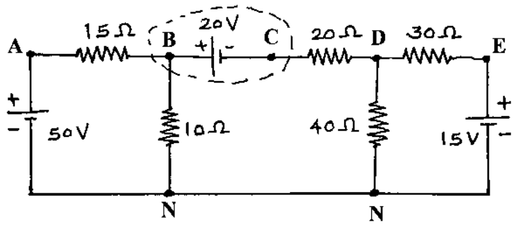
Let the nodes be A, B, C, D, E and reference node is N.

From the above circuit diagram we can write,

$$V_A = 50$$
$$V_E = 15$$
$$V_B - V_C = 20$$
$$\therefore V_C = V_B - 20$$

Only two unknown voltages are V_B and V_D.

Step II: Apply KCL at nodes with unknown voltages



Since there is a voltage source of 20V between nodes B and C, for writing KCL equations, let us treat nodes B and C with source as "Supernode", encircled by dotted line.

By KCL at this supernode, we can write

$$\frac{V_B - V_A}{15} + \frac{V_B}{10} + \frac{V_C - V_D}{20} = 0$$

$$\frac{V_B - 50}{15} + \frac{V_B}{10} + \frac{(V_B - 20) - V_D}{20} = 0$$

$$V_B\left[\frac{1}{15} + \frac{1}{10} + \frac{1}{20}\right] - \frac{50}{15} - \frac{20}{20} - V_D\left[\frac{1}{20}\right] = 0$$

$$\boldsymbol{(0.217)V_B - (0.05)V_D = 4.33} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(\boldsymbol{i})$$

1 mark for eq. (i)

By KCL at node D, we write

$$\frac{V_D - V_C}{20} + \frac{V_D}{40} + \frac{V_D - V_E}{30} = 0$$

$$\frac{V_D - (V_B - 20)}{20} + \frac{V_D}{40} + \frac{V_D - 15}{30} = 0$$

$$V_B\left[-\frac{1}{20}\right] + \frac{20}{20} - \frac{15}{30} + V_D\left[\frac{1}{20} + \frac{1}{40} + \frac{1}{30}\right] = 0$$

$$(-0.05)V_B + (0.1083)V_D = -0.5$$

$$\boldsymbol{(0.05)V_B - (0.1083)V_D = 0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(\boldsymbol{ii})$$

1 mark for eq. (ii)

Step III: Solving Simultaneous equations

Expressing eq. (i) and (ii) in matrix form,

$$\begin{bmatrix} 0.217 & -0.05 \\ 0.05 & -0.1083 \end{bmatrix}\begin{bmatrix} V_B \\ V_D \end{bmatrix} = \begin{bmatrix} 4.33 \\ 0.5 \end{bmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 0.217 & -0.05 \\ 0.05 & -0.1083 \end{vmatrix} = -0.0235 - (-0.0025) = -0.021$$

By Cramer's rule,

$$V_B = \frac{\begin{vmatrix} 4.33 & -0.05 \\ 0.5 & -0.1083 \end{vmatrix}}{\Delta} = \frac{(-0.469) - (-0.025)}{-0.021} = \frac{-0.444}{-0.021}$$

$$\boldsymbol{V_B = 21.143\ \textbf{volt}}$$

1 mark for stepwise solution for V_B and V_D

$$V_D = \frac{\begin{vmatrix} 0.217 & 4.33 \\ 0.05 & 0.5 \end{vmatrix}}{\Delta} = \frac{(0.1085) - (0.2165)}{-0.021} = \frac{-0.108}{-0.021}$$

$$\boldsymbol{V_D = 5.143\ \textbf{volt}}$$

Step IV: Solving for currents

Current in 40Ω resistor is given by,

$$I_{40} = \frac{V_D}{40} = \frac{5.143}{40} = \boldsymbol{0.1286\ A}$$

1 mark

Current in 10Ω resistor is given by,

$$I_{10} = \frac{V_B}{10} = \frac{21.143}{10} = \mathbf{2.1143\ A}$$ 1 mark

5 c) Use Norton's theorem to find the current through 3 Ω resistance, for the circuit shown in fig. no. 5

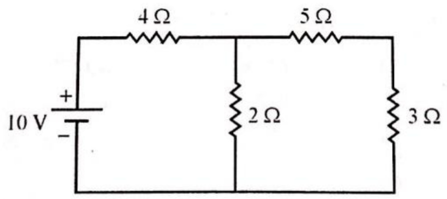
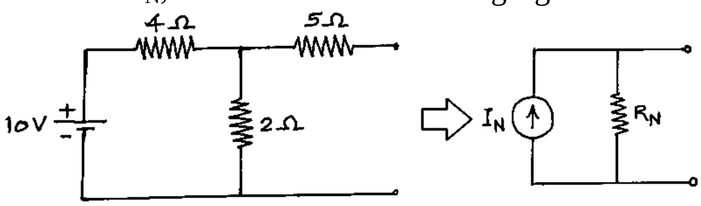


Fig. 5

Ans:

Solution by Norton's Theorem:

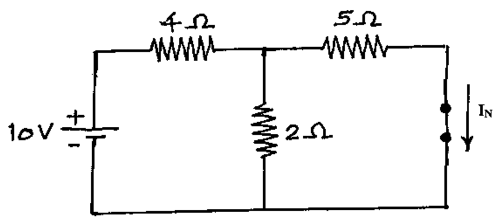
According to Norton's theorem, the circuit between load terminals excluding load resistance can be represented by simple circuit consisting of a current source I_N in parallel with a resistance R_N, as shown in the following figure.



1 mark

Determination of Norton's Equivalent Current Source (I_N):

Norton's equivalent current source I_N is the current flowing through a short-circuit across the load terminals due to internal sources, as shown in fig.(a).



(a)

marks for stepwise solution of I_N with circuit diagram

Total resistance across 10V source is,

$R = 4 + (5||2) = 4 + \frac{5\times2}{5+2} = 5.43\ \Omega$

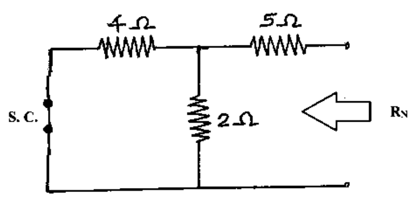
Therefore, current supplied by source,

$$I = \frac{V}{R} = \frac{10}{5.43} = 1.84\ A$$

The resistances 2Ω and 5Ω are in parallel. By current division, the current flowing through 5Ω is same as I_N.

$$\mathbf{I_N} = I\frac{2}{2+5} = (1.84)\frac{2}{7} = \mathbf{0.526\ A}$$

Determination of Norton's Equivalent Resistance (R_N):



(b)

1 mark for stepwise solution of R_N

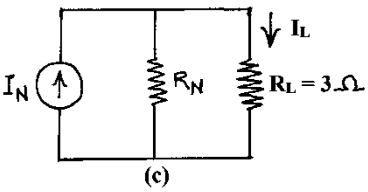
Norton's equivalent resistance is the resistance seen between the load terminals while looking back into the network, with internal independent voltage sources replaced by short-circuit and independent current sources replaced by open-circuit. Referring to fig.(b),

1 mark for equivalent

$$\mathbf{R_N} = 5 + (2||4) = 5 + \frac{2\times 4}{2+4} = \mathbf{6.33\ \Omega}$$

Determination of Load Current (I_L):
Referring to fig.(c), the load current is

$$\mathbf{I_L} = I_N\frac{R_N}{R_N + R_L} = 0.526\frac{6.33}{6.33+3} = \mathbf{0.357\ A}$$



circuit

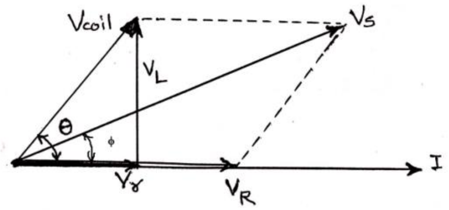
1 mark for I_L

6 **Attempt any <u>TWO</u> of the following:** **12**

6 a) Voltage across a coil is 146.2V and across series resistance is 150V, when they are connected across 220V, 50Hz supply. If supply current is 10 amp, find:

i) Resistance of coil
ii) Inductance of coil
iii) Power consumed by coil
iv) Power factor of total circuit

Ans:
Data given:
V_S = 220V, f = 50Hz, V_{Coil} = 146.2V, V_R = 150V, I = 10A



1 mark for phasor diagram

Referring to the phasor diagram above,

$$V_s = \sqrt{V_R^2 + V_{Coil}^2 + 2V_RV_{Coil}cos\theta}$$

$$\therefore cos\theta = \frac{V_s^2 - V_R^2 - V_{Coil}^2}{2V_RV_{Coil}} = \frac{(220)^2-(150)^2-(146.2)^2}{2(150)(146.2)} = 0.1032$$ ½ mark for θ

$\therefore$ Phase angle of circuit $\theta = \cos^{-1}(0.1032) = \mathbf{84.07°}$

$\therefore$ Voltage across resistance of coil, $V_r = V_{Coil}\cos\theta$ = (146.2)(0.1032) = 15.087 volt ½ mark

$\therefore$ Voltage across inductance of coil, $V_L = V_{Coil}\sin\theta$ = (146.2)sin(84.07°) = 145.42volt ½ mark

(i) **Resistance of Coil:**
Resistance of coil, $r = V_r / I$ = 15.087/10 = **1.5087Ω** ½ mark for r

(ii) **Inductance of Coil:**
Inductive Reactance of Coil, $X_L = V_L / I$ = 145.42/10 = 14.54Ω
$\therefore$Inductance of Coil, $L = X_L/(2\pi f)$ = 14.54/(2π×50) = **0.0462H** 1 mark for L

(iii) **Power consumed by coil:**
$P = I^2r = (10)^2(1.5087)$ = **150.87 watt** 1 mark for P

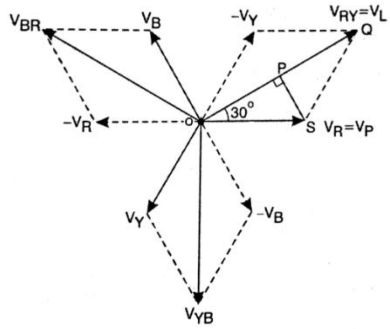
(iv) **Power factor of total circuit:**
Referring to the phasor diagram above,
$V_S\cos\phi = (V_r + V_R)$
$\therefore$Power factor of total circuit, $\cos\phi = (V_r + V_R)/ V_S$ = (15.087+150)/220 1 mark for total pf
$\therefore$ **cosϕ = 0.75 lagging**

6 b) In a 3 phase star connected system, derive the relationship $V_L = \sqrt{3}\ V_{ph}$.

Ans:

Relationship Between Line voltage and Phase Voltage in Star Connected System:

Let V_R, V_Y and V_B be the phase voltages.
V_{RY}, V_{YB} and V_{BR} be the line voltages.
The line voltages are expressed as:
$V_{RY} = V_R - V_Y$
$V_{YB} = V_Y - V_B$
$V_{BR} = V_B - V_R$



2 marks for phasor diagram

In phasor diagram, the phase voltages are drawn first with equal amplitude and displaced from each other by 120°. Then line voltages are drawn as per the above equations. It is seen that the line voltage V_{RY} is the phasor sum of phase voltages V_R and $-V_Y$. We know that in parallelogram, the diagonals bisect each other with an angle of 90°.

3 marks for stepwise explanation

Therefore in ΔOPS, ∠P = 90° and ∠O = 30°.

$$[OP] = [OS] \cos 30°$$

Since $[OP] = V_L/2$ and $[OS] = V_{ph}$

$$\therefore \frac{V_L}{2} = V_{ph}\cos 30°$$

$$V_L = 2V_{ph}\frac{\sqrt{3}}{2}$$

$$\mathbf{V_L = \sqrt{3}\, V_{ph}}$$

Thus **Line voltage = $\sqrt{3}$ (Phase Voltage)**

1 mark for final answer

6 c) State the Thevenin's theorem. Also write stepwise procedure for applying Thevenin's theorem to simple circuits.

Ans:

Thevenin's Theorem:

Any two terminal circuit having number of linear impedances and sources (voltage, current, dependent, independent) can be represented by a simple equivalent circuit consisting of a single voltage source V_{Th} in series with an impedance Z_{Th}, where the source voltage V_{Th} is equal to the open circuit voltage appearing across the two terminals due to internal sources of circuit and the series impedance Z_{Th} is equal to the impedance of the circuit while looking back into the circuit across the two terminals, when the internal independent voltage sources are replaced by short-circuits and independent current sources by open circuits.

2 marks for theorem

Stepwise Procedure for applying Thevenin's theorem to simple circuits:

Step I: Identify the load branch (R_L): It is the branch whose current is to be determined.

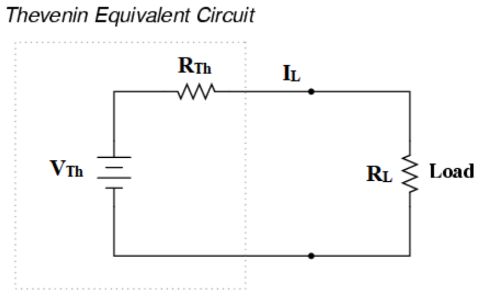
Step II: Calculation of V_{Th}: Remove R_L and find open circuit voltage across the load terminals A and B, which are now open due to removal of R_L.

Step III: Calculation of R_{Th}: It is the resistance between the open circuited load terminals A & B while looking back into the network with all independent voltage sources replaced by short-circuit & all independent current sources

4 marks for Stepwise procedure

replaced by open-circuit.

Step IV: Thevenin's equivalent circuit:



Step V: Determination of Load current:

$$I_L = V_{Th}/(R_{Th}+R_L)$$