Instructions –

(1) All Questions are *Compulsory*.

(2) Answer next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

1. Attempt any **FIVE** of the following: 10

   a) If \( f(x) = 3x^2 - 5x + 7 \), show that \( f(-1) = 3f(1) \)

   b) State whether the function \( f(x) = 3x^4 + x^2 + 5 - 3\cos x + 2\sin^2 x \) is even or odd.

   c) Find \( \frac{dy}{dx} \) if \( y = e^x \sin^{-1} x \)

   d) Evaluate \( e^{\int 2 \cdot \log x \, dx} \)

   e) Evaluate \( \int \sin^2 x \, dx \)

   f) Find the area under the curve \( y = x^2 \) from \( x = 0 \) to \( x = 3 \) with \( x \) axis.

   g) Express \( z = 1 - i \) in Polar form.
2. **Attempt any THREE of the following:**

   a) Find \( \frac{dy}{dx} \) if \( x^2 + y^2 = 4xy \)
   
   b) If \( x = a(\theta + \sin \theta), \ y = a(1 - \cos \theta) \)
      find \( \frac{dy}{dx} \) at \( \theta = \pi/2 \)
   
   c) Find radius of curvature of the curve \( \sqrt{x} + \sqrt{y} = 1 \) at \( \left( \frac{1}{4}, \frac{1}{4} \right) \)
   
   d) Find the maximum and minimum value of \( x^3 - 9x^2 + 24y \)

3. **Attempt any THREE of the following:**

   a) Find equation of tangent and normal to the curve \( 2x^2 - xy + 3y^2 = 18 \) at \( (3, 1) \)
   
   b) Find \( \frac{dy}{dx} \) if \( y = x^x + (\sin x)^x \)
   
   c) If \( y = e^{3\sec x} + 4\tan x \)
      find \( \frac{dy}{dx} \)
   
   d) Evaluate \( \int \frac{\sec^2 x}{(1 + \tan x)(3 + \tan x)} \, dx \)

4. **Attempt any THREE of the following:**

   a) Evaluate \( \int x \tan^{-1} x \, dx \)
   
   b) Evaluate \( \int \frac{dx}{4 + 5\cos x} \)
   
   c) Evaluate \( \int \frac{2x^2 + 5}{(x - 1)(x + 2)(x + 3)} \, dx \)
   
   d) Evaluate \( \int \frac{dx}{\sqrt{16 - 6x - x^2}} \)
   
   e) Evaluate \( \int_0^{\pi/2} \frac{dx}{1 + \cot x} \)
5. **Attempt any TWO of the following:**

a) Find the area between the curves $y = x$ and $y = x^2$

b) Attempt the following:
   
   (i) Find the order and degree of the differential equation
   $$\frac{d^2 y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$$

   (ii) Solve
   $$\frac{dy}{dx} + y \cot x = \csc x$$

c) If $L \frac{di}{dt} = 30 \cdot \sin (10 \pi t)$, find $i$ in terms of $t$, given that $L=2$ and $i=0$ at $t = 0$

6. **Attempt any TWO of the following:**

a) Attempt the following
   
   (i) Express $\frac{2 - \sqrt{3} i}{1 + i}$ in $x + iy$ form

   (ii) Find $L\{e^{-4t} t^2\}$

b) Find $L^{-1}\left\{ \frac{2s^2 - 4}{(s + 1)(s - 2)(s - 3)} \right\}$

c) Solve using Laplace transform $\frac{dx}{dt} + 2x = e^{-t}$ given that $x(0) = 2$