Q.1 Attempt any FIVE of the following 10 Marks

a) If \( f(x) = \log(\sin x) \), find \( f\left(\frac{\pi}{2}\right) \).

b) State whether the function \( f(x) = \frac{a^x + a^{-x}}{2} \) is even or odd.

c) Find \( \frac{dy}{dx} \) if \( y = e^x \cdot \tan^{-1} x \)

d) Evaluate \( \int e^{\log_e x} \, dx \)

e) Evaluate : \( \int \tan^2 x \, dx \)

f) Find the area bounded by the curve \( y = x \), \( X \)-axis & the ordinates \( x = 0 \), \( x = 2 \).

g) Express \( z = 1 + i \) in polar form.
Q.2 Attempt any THREE of the following 12 Marks

a) Find \( \frac{dy}{dx} \) if \( x^2 + y^2 + xy - y = 0 \). Find \( \frac{dy}{dx} \) at \((1,2)\)

b) If \( x = a \cos^3 \theta \) and \( y = b \sin^3 \theta \). Find \( \frac{dy}{dx} \) at \( \theta = \frac{\pi}{4} \)

c) An electrical pole wire near a factory hangs in the form of a curve \( y = \log(\sin x) \). Find the radius of curvature at \( x = \frac{\pi}{4} \)

d) If \( I_1 \) and \( I_2 \) be the currents and \( R_1 \) and \( R_2 \) be the two resistances in parallel to the total current \( I = I_1 + I_2 \) which is constant. Then the heat developed in a circuit is given by \( H = \frac{1}{t} \left\{ I_1^2R_1 + I_2^2R_2 \right\} \). Show that heat developed in a circuit is minimum if \( I_1R_1 = I_2R_2 \) where \( R_1, R_2, t \) are constants

Q.3 Attempt any THREE of the following 12 Marks

a) Find equation of tangent & normal to the curve \( y = x^3 - 2x^2 + 4 \) at \( x = 2 \).

b) Find \( \frac{dy}{dx} \) if \( y = x^x + (\sin x)^x \)

c) If \( y = e^{3\tan x + 4 \sec x} \), find \( \frac{dy}{dx} \)

d) Evaluate \( \int \frac{3 \tan^{-1} x}{1 + x^2} \, dx \)

Q.4 Attempt any THREE of the following 12 Marks

a) Evaluate \( \int \frac{dx}{\sqrt{13 - 6x - x^2}} \)

b) Evaluate: \( \int \frac{dx}{5 + 4 \cos x} \)

c) Evaluate \( \int x \sin^{-1} x \, dx \)

d) Evaluate \( \int \frac{x + 1}{x(x^2 - 4)} \, dx \)
e) Evaluate: \[ \int_{0}^{\pi/2} \frac{1}{1 + \tan x} \, dx \]

Q.5 Attempt any TWO of the following \hspace{1cm} 12 Marks

a) Find the area between the parabolas \( y^2 = 4x \) and \( x^2 = 4y \).

b) Attempt the following:

i) Find the order & degree of the differential equation \( \frac{3}{\sqrt{\frac{d^2 y}{dx^2}}} = \frac{dy}{dx} \)

ii) Solve: \( \frac{dy}{dx} + \frac{y}{x} = \sin x \)

c) The quantity of a charge of coulombs passes through a conducting wire during small interval of time \( t \) sec is given by \( \frac{dq}{dt} = i \) where \( i \) is current in ampere. If \( i = 10 \sin 100 \, t \) and that \( q = 0, t = 0 \) find the charge at time \( t \).

Q.6 Attempt any TWO of the following \hspace{1cm} 12 Marks

a) Attempt the following:

i) Express \( \frac{1+i}{2-i} \) in \( x + iy \) form

ii) Find \( L\{ e^{-3t} \, t^2 \} \)

b) Find \( L^{-1} \left\{ \frac{4s + 5}{(s - 1)^2 \cdot (s + 2)} \right\} \)

c) Solve the differential equation using Laplace transform:
\[ \frac{dq}{dt} + \frac{q}{Rc} = \frac{E}{R} \; ; \; q(0)=0 \]
Instructions:
1. All Questions are Compulsory.
2. Answer each next main Question on a new page.
3. Illustrate your answers with neat sketches wherever necessary.
4. Non Programmable pocket calculator is allowed.
5. Figures to the right indicate full marks.
6. Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Q.1 Attempt any FOUR of the following 08 Marks

a) If \( f(x) = x^2 + 4 \) find \( f(x + 1) - f(x - 1) \).

b) State with proof whether the function \( f(x) = x^3 - 3x + \sin x \) is even or odd.

c) Find \( \frac{dy}{dx} \) if \( y = (x^3 - 7x^2 + 3) \cdot (x^3 - 1) \)

d) Find \( \frac{dy}{dx} \) if \( x = at^2 \) and \( y = 2at \)

e) At what point of the curve \( y = 3x - x^2 \), the slope of tangent is \(-5\)?

f) Evaluate: \( \int e^{\log x} \, dx \)
Q.2 Attempt any **THREE** of the following 12 Marks

a) Find \( \frac{dy}{dx} \) if \( x^2 + y^2 + xy - y = 0 \) at \((1,2)\)

b) Find the equation of tangent & normal to the curve \( x^2 + 3xy + y^2 = 5 \) at the point \((1, 1)\).

c) Find the values of \( x \) for which the function is maximum and minimum if

\[
y = x^3 - \frac{15x^2}{2} + 18x.
\]

d) Evaluate

\[
\int \left\{ \frac{1}{1 + x^2} - x^3 + 5^x + \frac{1}{\sqrt{1-x^2}} \right\} \, dx
\]
Instructions:
1. All Questions are Compulsory.
2. Answer each next main Question on a new page.
3. Illustrate your answers with neat sketches wherever necessary.
4. Non Programmable pocket calculator is allowed.
5. Figures to the right indicate full marks.
6. Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Q.1 Attempt any **FOUR** of the following. **08 Marks**

a) Evaluate: \[ \int \frac{5 \log x}{x} \, dx \]

b) Evaluate: \[ \int \frac{1}{(x - 1)(x - 4)} \, dx \]

c) Evaluate: \[ \int_0^1 \frac{dx}{x^2 + 1} \]

d) Find the area bounded by the curve \( y = x \), X-axis & the ordinates \( x = 0, \ x = 2 \).

e) Find the order & degree of the differential equation \( \left( \frac{d^2 y}{dx^2} \right)^3 = \left( y + \frac{dy}{dx} \right)^8 \)

f) Find polar form of \( 1 + i \)
Q.2 Attempt any **THREE** of the following. 12 Marks

a) Evaluate: \( \int x \cdot \sin 2x \, dx \)

b) Solve: \( \cos^2 x \frac{dy}{dx} + y = \tan x \)

c) Find \( L \{e^{-3t}(2\cos 5t - 3\sin 5t)\} \)

d) Solve the differential equation using Laplace transform:

\[
\frac{dq}{dt} + \frac{q}{Rc} = \frac{E}{R} \quad ; \quad q(0) = 0
\]