## 22206

## 11920

3 Hours / 70 Marks
Seat No. $\square$
Instructions - (1) All Questions are Compulsory.
(2) Answer each next main Question on a new page.
(3) Figures to the right indicate full marks.
(4) Use of Non-programmable Electronic Pocket Calculator is permissible.
(5) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

## Marks

1. Attempt any FIVE of the following:
a) If, $f(x)=x^{2}-x+1$, then find $f(0)+f(3)$.
b) Show that, $f(x)=\frac{a^{x}+a^{-x}}{2}$ is an even function.
c) Find $\frac{d y}{d x}$, if $y=x^{5}+5^{x}+e^{x}+\log _{2} x$
d) Evaluate, $\int \frac{1}{1+\cos 2 x} d x$
e) Evaluate, $\int x \cdot e^{x} \cdot d x$
f) Find area bounded by the curve $y=x^{3}, x$-axis and the ordinate $x=1$ to $x=3$.
g) If a fair coin is tossed three times, then find probability of getting exactly two heads.
2. Attempt any THREE of the following:
a) Find $\frac{d y}{d x}$ if, $e^{x}+e^{y}=e^{x+y}$
b) If, $x=a \cos ^{3} \theta$ and $y=b \sin ^{3} \theta$. Find $\frac{d y}{d x}$ at $\theta=\frac{\pi}{3}$
c) A telegraph wire hangs in the form of $a$ curve $y=a \cdot \log \left[\sec \left(\frac{x}{a}\right)\right]$. Where $a$ is constant. Show that, radius of curvature at any point is $a \cdot \sec \left(\frac{x}{a}\right)$.
d) A beam is supported at the two ends and is uniformly loaded. The bending moment $M$ at a distance $x$ from the end is given by $M=\frac{W 1}{2} \times x-\frac{W}{2} \times x^{2}$. Find the point at which $M$ is maximum.
3. Attempt any THREE of the following:
a) Find the equation of tangent and normal to the curve $y=x^{2}$ at point $(-1,1)$
b) Find $\frac{d y}{d x}$ if, $y=x^{\sin x}$.
c) Find $\frac{d y}{d x}$ if, $y=\tan ^{-1}\left(\frac{x}{1+12 x^{2}}\right)$
d) Evaluate, $\int \frac{\left(\sin ^{-1} x\right)^{3}}{\sqrt{1-x^{2}}} d x$.
4. Attempt any THREE of the following:
a) Evaluate, $\int \frac{e^{x}(x+1)}{\cos ^{2}\left(x \cdot e^{x}\right)} d x$.
b) Evaluate, $\int \frac{d x}{5-4 \cos x}$.
c) Evaluate, $\int \tan ^{-1} x \cdot d x$.
d) Evaluate, $\int \frac{e^{x} \cdot d x}{\left(e^{x}-1\right)\left(e^{x}+1\right)}$.
e) Evaluate, $\int_{0}^{\pi / 2} \frac{1}{1+\tan x} d x$
5. Attempt any TWO of the following:
a) Find area bounded by the curve $y^{2}=4 x$ and $x^{2}=4 y$.
b) Attempt the following:
(i) Form a differential equation by eliminating arbitary constant if $y=\mathrm{A} \cdot \cos (\log x)+\mathrm{B} \sin (\log x)$.
(ii) Solve, $x\left(1+y^{2}\right) d x+y \cdot\left(1+x^{2}\right) d y=0$.
c) A particle starting with velocity $6 \mathrm{~m} / \mathrm{s}$. has an acceleration $\left(1-t^{2}\right) \mathrm{m} / \mathrm{s}^{2}$. When does it first come to rest? How far has it then travelled?
6. Attempt any TWO of the following:
a) (i) An unbiased coin is tossed 5 times. Find probability of getting three heads.
(ii) Fit a poissons distribution for the following observations.

| $x_{i}$ | 20 | 30 | 40 | 50 | 60 | 70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f i$ | 8 | 12 | 30 | 10 | 6 | 4 |

b) If $2 \%$ of the electric bulbs manufactured by a company are defective. Find the probability that in sample of 100 bulbs
(i) 3 are defective
(ii) At least two are defective.
c) In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5 . Assuming the distribution is to be normal,
(i) How many students score between 12 and 15.
(ii) How many students score above 18.

Given
Frequency 0 to $0.8=0.2881$
Frequency 0 to $0.4=0.1554$
Frequency 0 to $1.6=0.4452$.

