22201

12223

3 Hours / 70 Marks Seat No.

Marks

1. **Attempt any FIVE of the following:** 10

a) Define Odd and Even functions.

b) If $f(x) = 3x + a$ and $f(1) = 7$, find a and $f(4)$.

c) If $y = x^{10} + 10^x + e^x + \log x$ find $\dfrac{dy}{dx}$.

d) Evaluate $\int x \log x \, dx$

e) Evaluate $\int \dfrac{1}{1 - \cos 2x} \, dx$

f) Using integration, find the area of the region bounded by the lines $2y + x = 8$, x-axis and the lines $x = 2$ and $x = 4$.

g) State the Simpson's one-third rule of numerical integration.

Marks

2. **Attempt any THREE of the following:** 12

a) Find $\dfrac{dy}{dx}$ if $y = \log[\operatorname{cosec}x - \cot x]$.

b) Find $\dfrac{dy}{dx}$ if $x = \sec^2\theta$, $y = \tan^3\theta$ at $\theta = \dfrac{\pi}{3}$.

c) A beam is bent in the form of the curve $y = 2\sin x - \sin 2x$. Find the radius of curvature at $x = \dfrac{\pi}{2}$.

d) A telegraph wire hangs in the form of a curve $y = a \log \sec\left(\dfrac{x}{a}\right)$ where a is constant show that the curvature at any point is $\dfrac{1}{a}\cos\left(\dfrac{x}{a}\right)$.

3. **Attempt any THREE of the following:** 12

a) Find the equation of the tangent and normal to the curve $13x^3 + 2x^2y + y^3 = 1$ at $(1, -2)$.

b) Find $\dfrac{dy}{dx}$ if $y = x^{\sin x} + (\tan x)^x$.

c) If $y = \log(x\sin 2x)$ find $\dfrac{dy}{dx}$.

d) Evaluate $\displaystyle\int \dfrac{e^x(x+1)}{\sin^2(xe^x)}\,dx$.

4. **Attempt any THREE of the following:** 12

a) Evaluate $\displaystyle\int \dfrac{1}{9x^2 + 6x + 10}\,dx$

b) Evaluate $\displaystyle\int \dfrac{1}{3 + 2\sin x + \cos x}\,dx$

c) Evaluate $\displaystyle\int x^2 - e^{3x}\,dx$

d) Evaluate $\displaystyle\int \dfrac{\cos x}{(4 + \sin x)\,(3 + \sin x)}\,dx$

e) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sqrt[3]{\sec x}}{\sqrt[3]{\sec x} + \sqrt[3]{\operatorname{cosec}x}}\,dx$

Marks

5.   **Attempt any TWO of the following:**   **12**

a) Find the area cut off from the parabola $4y = 3x^2$ with the line $2y = 3x + 12$.

b) Attempt the following

   i)   Form the differential equation from the relation $y = Ae^{3x} + Be^{-3x}$ where A & B are arbitrary constant's.

   ii)   Solve $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$.

c) A right circular cone has height 9 cm and radius of the base 5 cm. It is inverted and water is poured into it. If at any instant the water level rises at the rate of $\left(\dfrac{\pi}{A}\right)$ cm/sec, where A is the area of the water surface at that instant, show that the vessel will be full in 75 seconds.

6.   **Attempt any TWO of the following:**   **12**

a) Attempt the following

   i)   Evaluate $\displaystyle\int_{3}^{8} \log_e x\, dx$ using Trapezoidal rule from the following data

x	3	4	5	6	7	8
$\log_e x$	1.0986	1.3863	1.6094	1.7918	1.9459	2.0794

   ii)   Apply Simpson's one-third rule to find $\displaystyle\int_{0}^{2} f(x)\, dx$ using the following data.

x	0	1.0	1.5	2.0
$f(x)$	1.1	2.4	5.7	8.1

b) Evaluate $\displaystyle\int_{0}^{\frac{\pi}{2}} \cos x\, dx$ using Simpson's $\dfrac{3}{8}$ rule with n = 8.

c) Evaluate $\displaystyle\int_{1}^{5} \dfrac{1}{x+2}\, dx$ using Simpson's one third rule. Divide the interval [1, 5] into 4 equal subintervals.

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