## 12223

3 Hours / 100 Marks Seat No.
$\square$ पाता

Instructions : (1) All Questions are compulsory.
(2) Illustrate your answers with neat sketches wherever necessary.
(3) Figures to the right indicate full marks.
(4) Assume suitable data, if necessary.
(5) Use of Non-programmable Electronic Pocket Calculator is permissible.
(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

## Marks

1. Attempt any TEN of the following :
(a) Find the point on the curve $y=x^{2}-3 x+3$, where the tangent is parallel to $x$-axis.
(b) Find equation of tangent to curve $y=e^{x}$ at origin.
(c) Evaluate $\int\left(\frac{3}{x}+\mathrm{a}^{x}-5 x^{4}+6 x\right) \mathrm{d} x$
(d) Evaluate $\int \frac{\cos (\log x)}{x} \mathrm{~d} x$
(e) Evaluate $\int_{0}^{\pi / 2} \sin 2 x \mathrm{~d} x$.
(f) Find the area bounded by the curve $y=x^{3}, x$-axis and the co-ordinate $x=2$ to $x=6$.
(g) Find the order and degree of the equation $\frac{1+\left(\frac{d y}{d x}\right)^{2}}{\frac{d^{2} y}{d x^{2}}}=5$.
(h) Form a differential equation for the following :

$$
\mathrm{y}=\mathrm{Ae}^{x}+\mathrm{Be}^{-x}
$$

(i) A die is tossed once. Find the probability of an event
(i) getting 3
(ii) not getting 4
(j) If two coins are tossed simultaneously then find the probabilities that
(i) Atleast one head
(ii) Both are heads
(k) Evaluate $\int_{0}^{\infty} \frac{1}{1+x^{2}} \mathrm{~d} x$
(l) Evaluate $\int_{0}^{1} x \cdot \mathrm{e}^{x} \mathrm{~d} x$.
2. Attempt any FOUR of the following :
(a) The slope of the curve $2 \mathrm{y}^{3}=\mathrm{ax}+\mathrm{b}$ at pt $(+1,-1)$ is same as the slope of $x+\mathrm{y}=0$. Find $\mathrm{a} \& \mathrm{~b}$.
(b) Evaluate:
$\int \frac{1}{4+5 \sin x} d x$
(c) Evaluate:
$\int x \cdot \tan ^{-1} x \mathrm{~d} x$
(d) Evaluate $\int \frac{\mathrm{d} x}{8-7 \sin ^{2} x}$
(e) Find the maximum \& minimum of the value of $x^{3}-18 x^{2}+96 x$.
(f) Evaluate $\int \frac{\sin x}{(1+\cos x)(2+\cos x)} \mathrm{d} x$

## 3. Attempt any FOUR of the following :

(a) Evaluate $\int_{\pi / 6}^{\pi / 3} \frac{\sqrt[3]{\sin x} \mathrm{~d} x}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}$
(b) Evaluate $\int_{0}^{\pi / 4} \log (1+\tan x) d x$
(c) Evaluate $\int \frac{\mathrm{d} x}{x^{2}-10 x+34}$
(d) Evaluate $\int \frac{1}{5+4 \cos 2 x} \mathrm{~d} x$
(e) A metal wire 100 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.
(f) Solve $\frac{d y}{d x}=\mathrm{e}^{x-y}+x \mathrm{e}^{-\mathrm{y}}$
4. Attempt any FOUR of the following :
(a) Evaluate $\int \frac{\log x}{x[2+\log x][3+\log x]} \mathrm{d} x$
(b) Evaluate $\int_{0}^{\pi / 2} \frac{\mathrm{~d} x}{4+5 \cos x}$
(c) Evaluate $\int_{1}^{3} \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}} \mathrm{~d} x$
(d) Find area of circle $x^{2}+y^{2}=36$ by using definite integral.
(e) Solve $\frac{\mathrm{dy}}{\mathrm{d} x}+\mathrm{y} \cot x=\cos x$.
(f) Solve $\left(x^{2}+1\right) \frac{d y}{d x}+y=e^{\tan ^{-1} x}$

## 5. Attempt any FOUR of the following :

(a) Solve $\left(3 x^{2}+6 x y^{2}\right) \mathrm{d} x+\left(6 x^{2} y+4 y\right) d y=0$
(b) Verify that $y=e^{m \cdot \sin ^{-1} x}$ is the solution of the differential equation $\left(1-x^{2}\right) \frac{d^{2} y}{d x^{2}}-x \frac{d y}{d x}-m^{2} y=0$
(c) Solve $\mathrm{y}^{2}+x^{2} \frac{\mathrm{dy}}{\mathrm{d} x}=x y \frac{\mathrm{dy}}{\mathrm{d} x}$
(d) In a certain examination 500 students appeared, mean score is 68 and S.D. is 8. Find the no. of students scoring less than 50 marks.
[Given: Area (2.25) = 0.4878]
(e) If $2 \%$ of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs 3 bulbs will be defective.
(f) Show that $y^{2}=a x^{2}$ is a solution of D.E.

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x\left(\frac{\mathrm{dy}}{\mathrm{~d} x}\right)^{2}-2 \mathrm{y} \frac{\mathrm{dy}}{\mathrm{~d} x}+\mathrm{a} x=0
$$

6. Attempt any FOUR of the following :
(a) In a college hostel there are 75 students like to drink tea, 40 like to drink coffee and 20 like neither tea nor coffee. Two students from this hostel come to canteen. Find the probability that both will order the same drink.
(b) Find the radius of curvature for the curve

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x=\mathrm{a} \cos ^{3} \theta, \mathrm{y}=\mathrm{a} \sin ^{3} \theta \text { at } \theta=\frac{\pi}{4}
$$

(c) Evaluate $\int \frac{e^{x}(x+1)}{\cos ^{2}\left(x e^{x}\right)} \mathrm{d} x$
(d) A card is drawn from a pack of 52 cards. Find the probability that a card is a diamond or a face card.
(e) An unbiased coin is tossed 5 times. Find the probability of getting
(i) Three heads
(ii) At least 4 heads
(f) The probability of getting an item defective is 0.005 . What is the probability that exactly 3 items in a sample of 200 are defective?
(Given : $\mathrm{e}^{-1}=0.3679$ )

