21222

3 Hours / 100 Marks   Seat No.

15 minutes extra for each hour

Instructions :   (1)   All Questions are *compulsory.*

(2)   Illustrate your answers with neat sketches wherever necessary.

(3)   Figures to the right indicate full marks.

(4)   Assume suitable data, if necessary.

(5)   Use of Non-programmable Electronic Pocket Calculator is permissible.

(6)   Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1.   Attempt any TEN of the following :   **20**

(a)   Find the point on the curve $y = 7x - 3x^2$ where tangent makes angle of 45°.

(b)   Find the slope of tangent to the curve $y = x^3$ at $x = 4$.

(c)   Evaluate $\int x(x - 1)^2 \, dx$.

(d)   Evaluate $\int \dfrac{1}{3x^2 + 4} \, dx$.

(e)   Evaluate $\int \dfrac{dx}{(x + 1)\,(x + 2)}$

(f)   Evaluate $\displaystyle\int_{4}^{9} \dfrac{dx}{\sqrt{x^3}}$

(g)   Evaluate $\displaystyle\int_{-1}^{1} \dfrac{1}{1 + x^2} \, dx$.

(h) Find order and degree of the D.E.

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 - 6y = 0$$

(i) Solve $e^y \frac{dy}{dx} = x^2$.

(j) Show that $(3x^2 - y)\,dx - x\,dy = 0$ is an exact differential equation.

(k) Find the probability of getting 9 with two dice.

(l) If a random variable has a Poisson distribution such that $P(2) = P(3)$. Find $P(5)$.

2. **Attempt any FOUR of the following :** 16

(a) Divide 100 into two parts such that their product is maximum.

(b) Find the equation of tangent and normal to the curve $y = x^3 - 2x^2 + 4$ at $x = 2$.

(c) Find the radius of curvature of the curve $y = x^3$ at point $(2, 8)$.

(d) Evaluate $\displaystyle\int \frac{\cos(\log x)}{x}\,dx$.

(e) Evaluate $\displaystyle\int \frac{dx}{5 + 4\cos x}$.

(f) Evaluate $\displaystyle\int \frac{2x^2 + 5}{(x-1)(x+2)(x+3)}\,dx$.

3. **Attempt any FOUR of the following :** 16

(a) Evaluate $\displaystyle\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+3}}\,dx$.

(b) Evaluate $\displaystyle\int_0^{\pi/2} \frac{1}{1 + \sqrt{\cot x}}\,dx$.

(c) Find the area of circle $x^2 + y^2 = 16$, by using integration.

(d) Solve $\dfrac{dy}{dx} = \cos(x + y)$.

(e) Solve $\sqrt{1-y^2}\, dx - \sqrt{1-x^2}\, dy = 0$.

(f) Solve $(x^2 + y^2)\, dx - 2xy\, dy = 0$.

4. Attempt any FOUR of the following : **16**

(a) Evaluate $\displaystyle\int_0^{\pi/4} \log(1 + \tan x)\, dx$.

(b) Evaluate $\displaystyle\int_1^3 \dfrac{\sqrt{x}}{\sqrt{4-x} + \sqrt{x}}\, dx$.

(c) Find the area bounded by the curve $y^2 = 4x$ and $x^2 = 4y$.

(d) Solve $(x + 1)\dfrac{dy}{dx} - y = e^x (1 + x)^2$.

(e) Solve $(x^2 + 6xy - y^2)dx + (3x^2 - 2xy + y^2)\, dy = 0$.

(f) Solve $\dfrac{dy}{dx} = e^{3x - 2y} + x^2\, e^{-2y}$.

5. Attempt any FOUR of the following : **16**

(a) Evaluate $\displaystyle\int x \cdot \tan^{-1} x\, dx$.

(b) Evaluate $\displaystyle\int_0^{\pi/2} \dfrac{\cos x}{4 - \sin^2 x}\, dx$.

(c) Find the area of ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$, by using integration.

(d) A box contains 10 red, 5 white, 5 black balls, two balls drawn at random. Find the probability that they are not of the same colour.

    (e)   From 20 tickets marked 1 to 20, one ticket is drawn at random. Find the probability that marked with multiple of 3 or 5.

    (f)   In 200 sets of tosses of 5 fair coins in how many ways you can expect

        (i)   atleast two heads.

        (ii)   at the most two heads.

6.   **Attempt any FOUR of the following :**   **16**

    (a)   Find the value of x for which function is maximum and minimum if

$$y = x^3 - \frac{15x^2}{2} + 18x.$$

    (b)   Show that equation of tangent to $\left(\dfrac{x}{a}\right)^m + \left(\dfrac{y}{b}\right)^m = 2$ at the point (a, b) is

$$\frac{x}{a} + \frac{y}{b} = 2.$$

    (c)   Evaluate $\displaystyle\int \frac{dx}{\sqrt{x^2 - 3x + 4}}$.

    (d)   If 2% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs

        (i)   3 bulbs are defective.

        (ii)   at the most two bulbs will be defective.

    (e)   An unbiased coin is tossed 5 times. Find the probability of getting 3 heads.

    (f)   In a sample of 1000 students the mean of certain test is 14 and standard deviation is 2.5 assuming the distribution to be normal. Find how many score above 18.

        (Given area between z = 0 and z = 1.6 is 0.4452).

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