SUMMER– 18 EXAMINATION

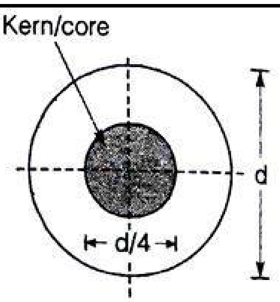
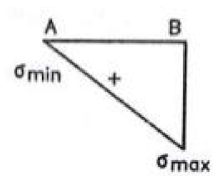
Subject Name: THEORY OF STRUCTURES **Model Answer** Subject Code: 17422

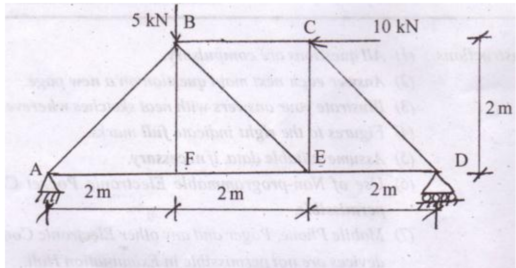
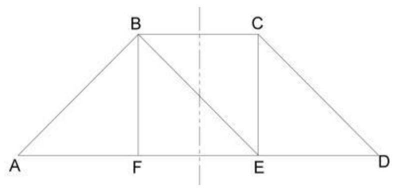
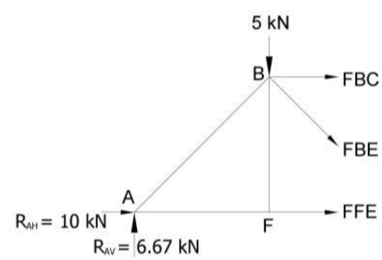
Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answers	Marking Scheme
Q.1	(A) a) Ans	***Attempt any six of the following:*** ***Define direct stress and bending stress.*** **Direct stress:** Direct stress is defined as the ratio of direct load to cross section area. It of compressive nature. **Bending stress:** The stress developed across the cross section due to bending moment is called bending stress. It may compressive or tensile in nature.	***(12)*** 01 M 01 M
Q.1	(A)b) Ans	***Define slope and deflection of beam.*** **Slope of beam:** The slope at any point on the elastic curve of the beam is defined as the angle in radians that the tangent at that point makes with the original axis of the beam. **OR** The angle made by a tangent at a point of a deflected beam with its neutral surface of loaded beam is called slope **Deflection of beam:** The deflection at any point on the axis of the beam is the (vertical) distance between its positions before and after loading. **OR** When a beam is loaded, it will deflect from its original position in the direction perpendicular to its longitudinal axis. This displacement of beam measured from its neutral axis from unloaded condition to loaded condition of beam is known as deflection of beam.	 01 M 01 M
Q.1	(A)c) Ans	***A cantilever beam of span L carries point load W at free end state the slope and deflection at free end in terms of EI.*** Slope at the free end **$\theta = WL^2/2EI$** Deflection at free end **$y = WL^3/3EI$** W: Point load acting at free end of cantilever. E: Modulus of Elasticity. L- Span of the cantilever beam. I - Moment of Inertia of beam.	 01 M 01 M

.1	(A)d) Ans	***State the situations where Macaulay's method is used to find slope and deflection.*** This method is convenient 1. When beam carries several point loads. 2. When u.d.l. acts over the entire span with or without point loads. 3. When u.d.l. starts from an intermediate point but extends up to one of the ends of the beam.	Any two 01 M for each
Q.1	(A)e) Ans	***Define fixed beam with sketch.*** The beam whose end supports are such that the slopes remain zero is called fixed beam **OR** A beam whose ends are firmly built in the support like wall, pillar or any other structure, such beams are called as fixed beam 	01 M 01 M
Q.1	(A)f) Ans	***Define distribution factor.*** The distribution factor for a member at a joint is the ratio of stiffness factor for that member and the total stiffness of all the members meeting at a joint. **OR** It is the ratio of moment shared by one of the members at joint with total moment applied at joint.	02 M
Q.1	(A)g) Ans	***State types of portal frames.*** **Types of portal frames:** 1. Symmetrical portal frame (frame having identical supports, equal column length, symmetrical loading, same MI and same modulus of elasticity.) 2. Unsymmetrical portal frame ((frame having different supports, column length, loading, same MI and same modulus of elasticity.)	01 M 01 M
Q.1	(A)h) Ans	***Define perfect and imperfect frames.*** **Perfect frame**: It is the simple frame in which number of joints (j) and number of members (m) satisfies the equation m = 2j – 3. Such frames are internally determinate i.e. can be analysed by using basic equations of equilibrium ($\Sigma M_A = 0$, $\Sigma F_x=0$ and $\Sigma F_y = 0$). **Imperfect frame**: It is the simple frame in which number of joints (j) and number of members (m) does not satisfy the equation m = 2j – 3. Such frames are internally indeterminate/redundant or deficient. If m > 2j – 3; then frame is called as indeterminate/redundant frame and cannot be analysed by using basic equations of equilibrium ($\Sigma M_A = 0$, $\Sigma F_x = 0$ and $\Sigma F_y = 0$). If m < 2j – 3; then frame is called as deficient frame and it is unstable frame.	01 M 01 M
Q.1	(B) a) Ans	***Attempt any two of the following:*** ***Define core of the section and derive the equation for core of the section for circular section.*** The centrally located portion of a section within the load line falls so as to produce only compressive stress is called core of the section.	*(08)* 02 M

		 **Circular Column** For circular section having diameter 'd' M = Bending Moment acting on column =P x e P= load on column (KN) e = eccentricity of column (mm) y_{max} = Max. centroidal distance = d/2; Z = Section modulus Z = Zxx= Zyy = I/ymax= ($\pi d^4/64$) /(d/2) = $\pi d^3/32$ Now, for no tension condition; This limiting eccentricity when load 'P' act anywhere form center. $e \le Z/A$ $e \le (\pi d^3/32) / (\pi d^2/4)$ $e \le d/8$ e_{max}= d/8 $2e_{max}$= 2 x d/8 $2e_{max}$= d/4 For no tension condition, the load must lie within a circle of diameter d/4 as shown in figure.	02 M
Q.1	(B)b) Ans	***State the condition of no tension at base and draw stress distribution for zero tension condition.*** When direct stress is greater than or equal to bending stress so as to avoid tensile stresses and creates only compressive stress, this condition is known as no tension condition. Or zero stress condition as the minimum stress is zero. If σ_o= direct stress (N/mm^2) σ_b= bending stress (N/mm^2) Then or no tension condition ($\sigma_o = \sigma_b$) σ_{max} = maximum stress at the base (N/mm^2)= $\sigma_o + \sigma_b = 2\sigma_o$ σ_{min} = minimum stress at the base (N/mm^2)= $\sigma_o - \sigma_b = 0$ 	02 M 02 M

Q.1	(B)c)	***Find the forces in the members of BC, BE and FE of the frame shown in fig. 1 using method of section.***  Figure 1	
	Ans	Calculation of reactions: R_{DV} = (5 x 2 – 10 x 2) / 6 = - 1.67 kN i.e. 1.67 kN downwards R_{AV} = 5 + 1.67 = 6.67 kN upwards R_{AH} = 10 kN towards right.  Consider section as shown in fig. and considering left part of the section. Assuming all forces tensile in nature. 1. Taking moment about joint B ($\Sigma M_B = 0$) - 10 x 2 + 6.67 x 2 – F_{FE} x 2 = 0 **F_{FE} = - 3.33 kN; i.e. 3.33 kN (Compressive)** 2. Taking moment about joint E ($\Sigma M_E = 0$) - 5 x 2 + 6.67 x 4 + F_{BC} x 2 = 0 **F_{BC} = - 8.34 kN; i.e. 8.34 kN (Compressive)** 3. $\Sigma F_Y = 0 = -5 + 6.67 - F_{BE}\cos 45^0$ **F_{BE} = 2.36 kN (Tensile)**  	01 M 01 M for each mamber

Q.2 ***Attempt any four of the following:*** ***(16)***

a) ***A rectangular column 350 mm wide and 250 mm thick carries an axial load of 225 kN and a clockwise moment of 3.5 kN- m in plane bisecting 250 mm side. Calculate the resultant stresses induced at the base. Draw stress distribution diagram.***

Ans

b= 250mm d= 350mm

P= 225KN= 225 x 10^3 N — 04 M

M= 3.5 KN-m = 3.5 x 10^6 N.mm

To calculate σ_{max} and σ_{min}

Area A = b x d= 350 x 250 = 8.75x 10^4 mm^2

σ_o = P/A = 225 x 10^3/8.75x 10^4 = 2.57 N/mm^2 (compressive)

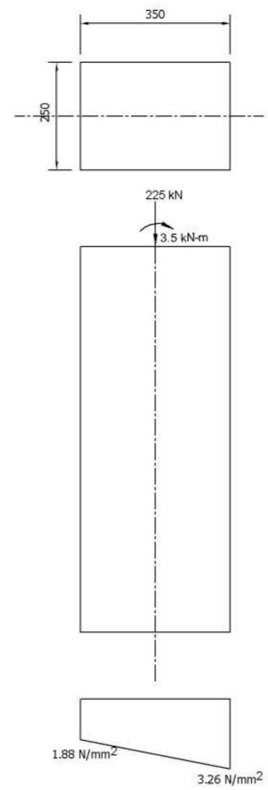
σ_b = M/ Z_{yy} = [M/ (bd^2/ 6)]

= (3.5 x 10^6) / [(250 x 350^2) / 6] = 0.69 N/mm^2

resultant stresses (σ_{max} and σ_{min})

σ_{max} = σ_o + σ_b = 2.57 + 0.69 = 3.26 N/mm^2 (compressive)

σ_{min} = σ_o - σ_b = 2.57 – 0.69 = 1.88 N/mm^2 (compressive)



Q.2 b) ***A cast iron column, 300 mm external diameter and 200 mm internal diagram carries a vertical compressive load of 250 kN. Find the maximum allowable eccentricity for this load for no tension condition.***

Ans

External diameter D= 300mm

Internal diameter d= 200 mm

Area =[π(D^2 – d^2)/4] = 3.927 x 10 ^{4}mm^2 — 01 M

Z_{yy} = [π(300^4-200^4)/64]/(300/2) = 2127120 mm^3

Direct stress

σ_o = P/A= 250 x 10^3/3.927x 10^4 = 6.36 N/mm^2 (compressive) — 01 M

σ_b = (P.e)/ Z_{yy} = M/ Z_{yy} = M/{[π(D^4-d^4)/64]/(D/2)} =(250 x 10^3 x e) /2127120 N/mm^2 — 01 M

σ_b = 0.0.11 e

for no tension condition

σ_o = σ_b

6.36 = 0.11 e

e = 54.11mm — 01 M

The maximum allowable eccentricity for this load will be 54.11 mm.

Q.2 c) ***Find maximum and minimum stress intensities induced on the base of masonry wall 12m high, 6 m wide and 1.5m thick subjected to a horizontal wind pressure of 1:2 kN/m^2 acting on 6 m side. The density of wall material is 22 kN/m^3.***

Ans

H= 12m, b= 6m, t= 1.5m, p= 1.2 KN/m^2, ρ= 22KN/m^3

To calculate:- maximum and minimum stress intensities (σ_{max} and σ_{min})

1. Direct stress σ_o = H x ρ = 12 x 22= 264 kN/m^2 — 01 M

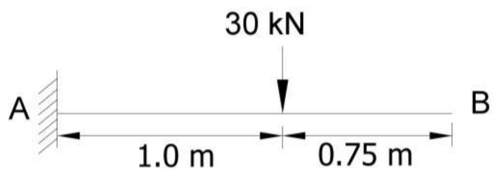
2. Total wind load (P)= p x projected area = p x b x H= 1.2 x 6 x 12 = 86.4 kN
3. Moment of P about the base = (M)= P x (H/2) = 518.4 kN.m
4. Section modulus of the base section about the axis of bending (Z)= [(b x t^2)/6]= (Z) = 2.25m^3
5. Bending stress= σ_b= M/Z= 230.4 kN/m^2 — 01 M

σ_{max} = maximum stress at the base = $\sigma_o + \sigma_b =$ 494.4 kN/m^2 (compressive) — 01 M

σ_{min} = minimum stress at the base = $\sigma_o - \sigma_b =$ 33.6 kN/m^2 (compressive) — 01 M

Q.2 d) ***A cantilever beam 150 mm wide and 225 mm deep projects 1.75 m out of wall and carries point load of 30 kN at a distance 1m from the fixed end. Find the deflection of cantilever at the free end. Take E = 200 kN/m².***

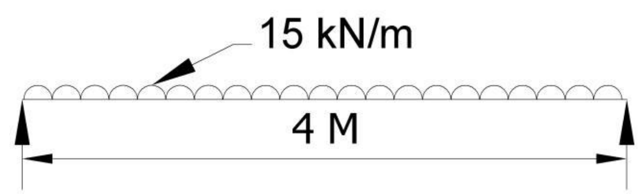
Ans



L_1= 1.0 m, L_2 = 0.75 m b= 150mm, d= 225mm,
W= 30 KN E= 200 kN/m^2
To calculate :- deflection at free end (y)
I=I_{xx} =(b x d^3)/12 = 150 x 225^3 / 12 = 1.423 x 10^8 mm^4 = 1.423 x 10^{-4} m^4 — 01 M
y = {[(wL_1^3)/3EI]+ [((wL_1^2)/2EI) x (L_2)]} — 02 M
= {(30 x 1^3)/(3 x 2 00 x 1.42 x 10^{-4})+((30 x 1^2)/(2 x 2 00 x 1.42 x 10^{-4}))x0.75)}
= 352.11 +396.13 =748.24 m
y= 748.24 m — 01 M
The deflection of cantilever at the free end will be 748.24m.
Note: Value of E given is very less. But if students take E = 200 kN/mm² then y = 0.75 mm.

Q.2 e) ***A simply supported beam of span 4 m carries a UDL of 15 kN/m over entire span. Find the deflection at mid span and slope at the ends. Ixx = 2 x10⁸mm⁴, E = 2 x10⁵ N/mm².***

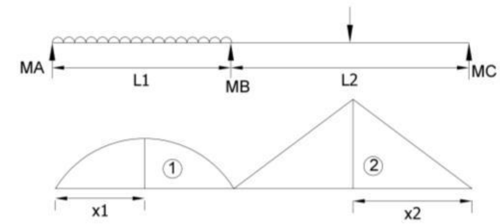
Ans



EI = 2 x 10^8 x 2 x 10^5 = 4 x 10^{13} N-mm^2 = 4 x 10^4 kN-m^2
$\theta_a = \theta_b$ = (WL^3)/ 24EI = (15 x 4^3)/ (24 x 4 x 10^4) = 1 x 10^{-3} rad. — 02 M
Y at mid span = ymax = 5 WL^4/ 384EI = (5 x 15 x 4^4) / (384 x 4 x 10^4)
= 1.25 x 10-3 m = 1.25 mm — 02 M

Q.2 f) State Clapeyron's theorem and also write the Clapeyron's three moment theorem for beam with different moment of inertial giving meaning of each term.

Ans Clapeyrons theorem: For two span continuous beam having uniform moment of inertia supported at A, B, and C and subjected to any external loading, the support moments M_A, M_B and M_C at the supports A, B and C respectively are given by the relation, — 02 Marks
M_A x L_1 + 2M_B(L_1 + L_2) + M_C x L_2 = – [(6 x a_1 x $\bar{x}_1$/L_1) + (6 x a_2 x $\bar{x}_2$/L_2)]



If moment of inertia is not constant then Clapeyrons theorem can be stated in form of following equation.

$M_A \times (L_1/I_1) + 2M_B[(L_1/I_1) + (L_2/I_2)] + M_C \times (L_2/I_2) = -[(6 \times a_1 \times x_1/L_1I_1) + (6 \times a_2 \times x_2/L_2I_2)]$

Where,

L_1 & L_2 are length of span AB & BC resp. — 01 Mark

I_1 & I_2 are Moment of inertia of span AB & BC resp.

a_1 & a_2 are area of simply supported BMD of span AB & BC resp.

x_1 & x_2 are distances of centroid of simply supported BMD from A & C resp. — 01 Mark

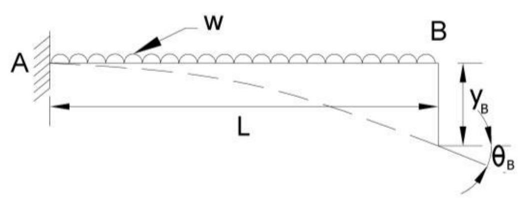
Q.3 ***Attempt any four of the following:*** ***(16)***

a)i) ***A cantilever beam of span L carries UDL over entire span. Write the expression to find slope and deflection at free end. Draw deflected shape.***

Ans

θ_B = Slope at free end = $w \times L^3/6EI$ -- 1/2 M

y_B = Deflection at free end = $w \times L^4/8EI$ --- 1/2 M



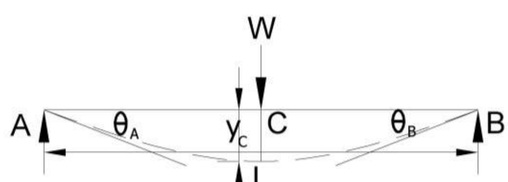
01 M

Q.3 a)ii) ***A simply supported beam of span L carries central point load. Write the expression to find slope at ends and deflection at centre. Draw deflected shape.***

Ans

$\theta_A = \theta_B$ = Slope at ends = $W \times L^2/16EI$ --- 1/2 M

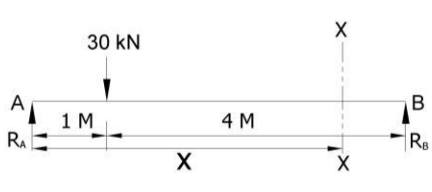
y_C = Deflection at center = $W \times L^3/48EI$ -- 1/2 M



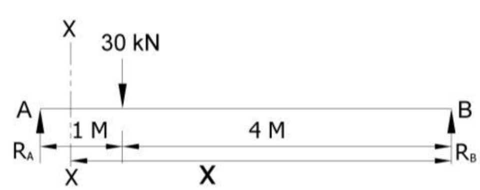
01 M

Q.3 b) ***A simply supported beam of span 5m carries a point load of 30 kN at 1m from left support. Calculate the deflection at mid span in terms of E I .Use Macaulay's Method.***

Ans



OR



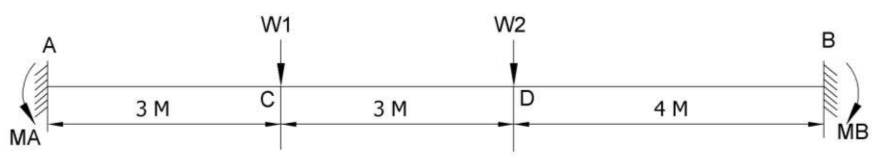
Reactions-

$R_A = 30 \times 4 / 5 = 24$ kN. $R_B = 30 - 24 = 6$ kN — 1/2 Mark

Taking section X-X at distance 'X' from A	Taking section X-X at distance 'X' from B	Marks
$M_x = 24 \times X - 30(X - 1)$	$M_x = 6 \times X - 30(X - 4)$	01 Mark
$EId^2y/dx^2 = - Mx$	$EId^2y/dx^2 = - Mx$	
$= - 24 \times X + 30(X - 1)$	$= - 6 \times X + 30(X - 4)$	
Integrating	Integrating	
$EIdy/dx = (- 24 \times X^2)/2 + [30(X - 1)^2]/2 + C_1$	$EIdy/dx = (- 6 \times X^2)/2 + [30(X- 4)^2]/2 + C_1$	
Integrating	Integrating	
$EIy = (- 12 \times X^3)/3 + [15(X - 1)^3]/3+C_1X + C_2$	$EIy = (- 3 \times X^3)/3 + [15(X- 4)^3]/3+C_1X + C_2$	
At $X = 0$; $y = 0$ in EIy eq^n.	At $X= 0$; $y = 0$ in EIy eq^n.	
$0 = 0 + C_2$	$0 = 0 + C_2$	
$C_2 = 0$	$C_2 = 0$	
At $X = 5$; $y = 0$ in EIy eq^n.	At $X = 5$; $y = 0$ in EIy eq^n.	
$0 = (- 4 \times 5^3) + [5 \times (5 - 1)^3] + C_1 \times 5 + 0$	$0 = (- 1 \times 5^3) + [5(5 - 4)^3] + C_1 \times 5 + 0$	
$C_1 = 36$	$C_1 = 24$	1/2 Mark
Hence $C_1 = 36$ and $C_2 = 0$	Hence $C_1 = 24$ and $C_2 = 0$	
Deflection equation-	Deflection equation-	
$y =(1/EI)[(- 4 \times X^3) + 5 \times (X - 1)^3+36X]$	$y =(1/EI)[(- 1 \times X^3)+5(X- 4)^3+24X]$	1/2 Mark
For deflection at mid span	For deflection at mid span	
Put $X = 2.5$ in eq^n.	Put $X= 2.5$ in eq^n.	
$y_c = (1/EI)[(- 4 \times 2.5^3)+5 \times (2.5 - 1)^3+36 \times 2.5]$	$y_c = (1/EI)[(- 1 \times 2.5^3)+ 0 +24 \times 2.5]$	01 M
$= 44.375 / EI$	$= 44.375 / EI$	1/2 M

Q.3 c) ***A fixed beam has span 10m carries two point loads W_1 and W_2 at 3 m and 6m from left hand support respectively. If fixed end moment at left hand support is 1.25 times that of right hand support. Find the ratio of $W_1$and W_2.***

Ans



$M_A = [(W_1 \times 3 \times 7^2) / 10^2] + [(W_2 \times 6 \times 4^2) / 10^2]$

$= (147W_1 + 96W_2) / 10^2$ — 01 M

$M_B = [(W_1 \times 7 \times 3^2) / 10^2] + [(W_2 \times 4 \times 6^2) / 10^2]$

$= (63W_1 + 144W_2) / 10^2$ — 01 M

But $M_A = 1.25M_B$

$(147W_1 + 96W_2) / 10^2 = 1.25(63W_1 + 144W_2) / 10^2$ — 01 M

Solving equation, Ratio $W_1 / W_2 = 1.23$ — 01 M

Q.3 d) ***State the advantages and disadvantages of fixed beam.***

Ans

Advantages of fixed beam: - (1/2 M for each)

1. The beam is stiffer and stronger.
2. The bending moment at center of span is reduced.
3. The deflection at center of span is reduced.
4. The slopes at ends of beam are zero.

Disadvantages of fixed beam: -

1. The slight sinking of support induces additional moment at each end.

<table>
<tr><td></td><td></td><td>2. Temperature stresses are induced due to variation in temperature.
3. Extra care has to be taken to achieve 100% fixity at ends.
4. Frequent fluctuations in loading due to moving loads are likely to disturb end fixity.</td><td>1/2 M for each</td></tr>
<tr><td>Q.3</td><td>e)
Ans</td><td>State the assumptions made in analysis of simple frames.
1. The ends of members are pin jointed.
2. The loads act only at joints.
3. Self-weight of members are neglected.
4. Members have uniform cross section throughout the length of member.</td><td>01 M for each</td></tr>
<tr><td>Q.3</td><td>f)

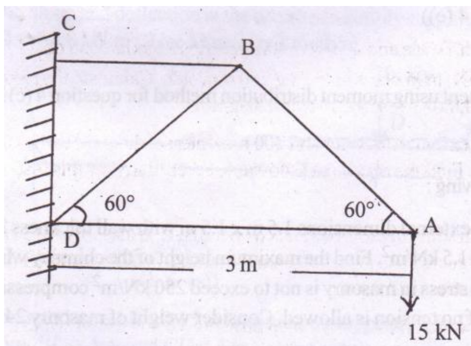
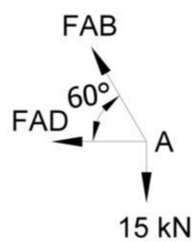
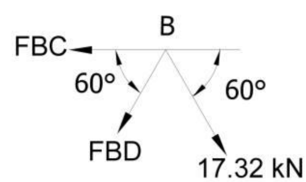
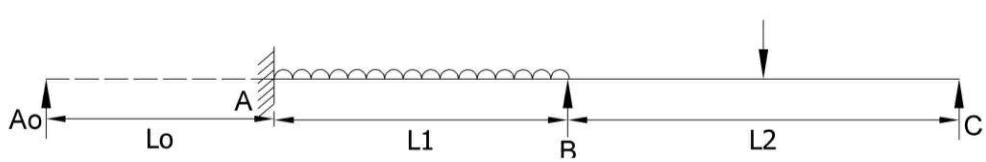
Ans</td><td>A cantilever truss of 3 m span is loaded as shown in Figure 2. Find the forces in the members AB, BC, BD and AD using method of joints.


Figure 2


Joint A:
$\Sigma F_y = F_{AB}\sin60 - 15 = 0$
$F_{AB} = 15 / \sin60 =$ 17.32 kN (Tensile)
$\Sigma F_X = - F_{AB}\cos60 - F_{AD} = 0$
$F_{AD} = - 17.32\cos60 = -8.66$ kN i.e. 8.66 kN (Compressive)
Joint B:


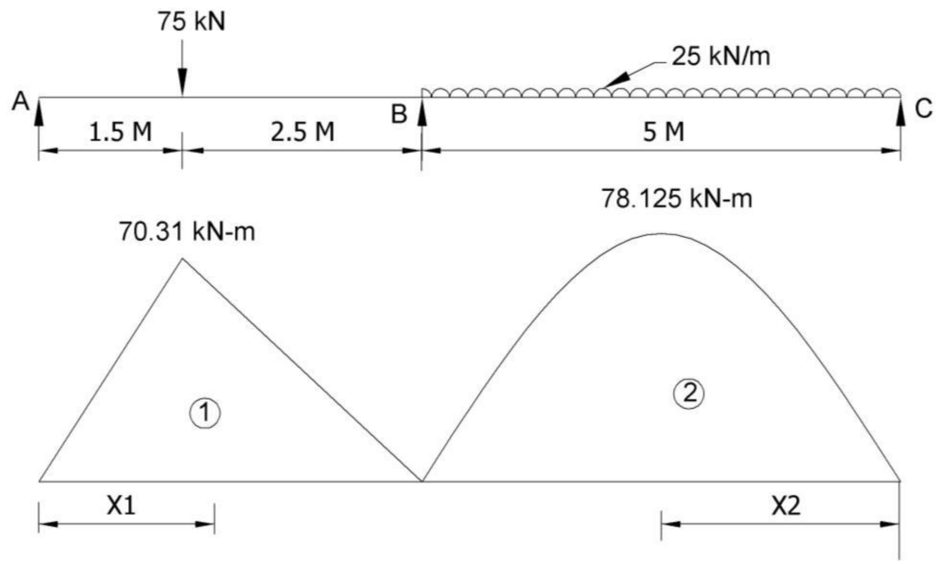
$\Sigma F_y = - F_{BD}\sin60 - 17.32\sin60 = 0$
$F_{BD} = - 17.32$ kN = i.e. 17.32 kN (Compressive)
$\Sigma F_X = - F_{BD}\cos60 - F_{BC} + 17.32\cos60 = 0$
$= - (-17.32)\cos60 - F_{BC} + 17.32\cos60 = 0$
$F_{BC} =$ 17.32 kN (Tensile)</td><td>02 M

02 M</td></tr>
<tr><td>Q.4</td><td>
a)
Ans</td><td>Attempt any four of the following:
Explain the concept of zero span in case of three moment theorem with sketch.
When the ends of continuous beam are fixed, then an imaginary span is considered to the left or right of the fixed support as the case may be and Clapeyrons theorem is applied to the imaginary span and its adjacent span as per regular procedure.
If left end is fixed then consider imaginary span left of this support and If right end is fixed then consider imaginary span on right side of that support.
Clapeyrons theorem is applied as below.


A_0-A is imaginary span left to fixed end A.</td><td>(16)

02 M

01 M</td></tr>
</table>

		For span A_0-A and AB $M_{A0} \times L_0 + 2M_A(L_0 + L_1) + M_B \times L_1 = -[(6 \times a_0 \times x_0/L_0) + (6 \times a_1 \times x_1/L_1)]$ $0 + 2M_A(L_1) + M_B \times L_1 = -[0 + (6 \times a_1 \times x_1/L_1)]$ Where M_{A0}, L_0 and x_0 are terms related to imaginary span.	01 M
Q.4	b) Ans	***Define continuous beam and state the effect of continuity in case of continuous beam.*** Definition: - It is defined as the beam which is supported over more than two supports. Effect of continuity: - 1. Deflection of entire beam reduces. 2. Slope on both sides of intermediate support is same. 3. Moments are developed at intermediate supports. 4. Sagging bending moments are developed at mid span and hogging bending moment developed at intermediate supports.	02 M 02 M
Q.4	c) Ans	***Calculate the support moment of continuous beam simply supported at A, B and C. Span AB =4 m and span BC =5m (i) Span AB carries point load of 75 kN at 1.5m from support A (ii) Span BC carries a UDL of 25 kN/m. Use three moment theorem.***  S.S.B.M. under 75 kN load = 75 x 1.5 x 2.5 / 4 = 70.31 kN-m S.S.B.M. at mid span of BC = $25 \times 5^2 / 8$ = 78.125 kN-m $A_1 = 0.5 \times 4 \times 70.31 = 140.625$ $X_1 = (4 + 1.5)/3 = 1.83$ m $A_2 = 2 \times 5 \times 78.125 / 3 = 260.41$ $X_2 = 2.5$ m $M_A = M_B = 0$ End simple supports. Applying theorem of three moment for span AB and BC- $M_A \times L_1 + 2M_B(L_1 + L_2) + M_C \times L_2 = -6 [(A_1 \times X_1 / L_1) + [(A_2 \times X_2 / L_2)]$ $0 + 2M_B(4 + 5) + 0 = -6 [(140.625 \times 1.83 / 4) + [(260.41 \times 2.5 / 5)]$ $18M_B = -6 (64.33 + 130.205)$ $= -1167.23$ $M_B = -64.84$ kN-m i.e 64.84 kN-m (Hogging)	01 M 01 M 01 M 01 M
Q.4	d) Ans.	***Explain the concept of stiffness factor and carry over moment.*** **Stiffness factor: -** It is the moment required at the simply supported end of beam to produce unit rotation at the end without translation of either ends of member. **Carry over factor: -** It is defined as moment induced at the fixed end of a beam by the action of the moment applied at the other simply supported or hinged end. The	02 M 02 M

moment induced at the fixed end is half of the moment applied at the simply supported end.

Q.4 e) ***Calculate the distribution factor at joint 0 for joint as shown in figure 3.***

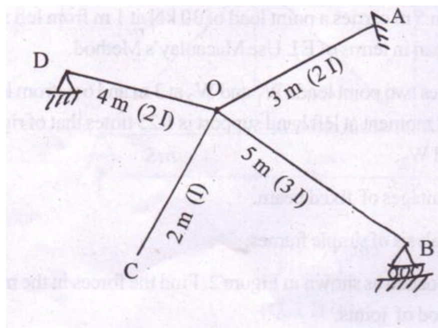


Figure 3

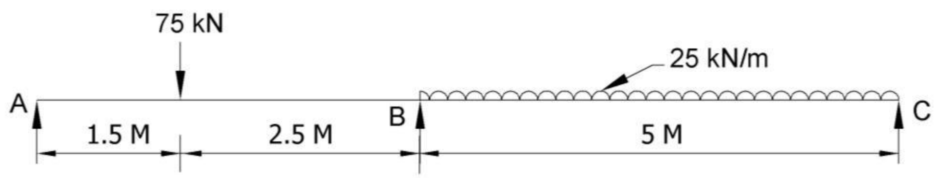
Ans.

Joint	Member	Stiffness k	Σk	d = k/ Σk
O	OA	4 x 2EI/3 = 2.67EI	5.97EI	2.67 / 5.97 = 0.446
	OB	3 x 3EI / 5 = 1.8EI		1.8 / 5.97 = 0.302
	OC	0		0
	OD	3 x 2EI / 4 = 1.5EI		1.5 / 5.97 = 0.252

01 M for each

Q.4 f) ***Calculate the support moment using moment distribution method for question 4 (C) having same M.I.***

Ans.



M_{AB} = - 75 x 1.5 x 2.5^2 / 4^2 = - 43.94 kN-m M_{BA} = 75 x 2.5 x 1.5^2 / 4^2 = 26.36 kN-m
M_{BC} = - 25 x 5^2 / 12 = - 52.08 kN-m M_{CB} = 25 x 5^2 / 12 = 52.08 kN-m — 01 M

Joint	Member	Stiffness k	Σk	d = k/ Σk
B	BA	3 x EI/4 = 0.75EI	1.35EI	0.75 / 1.35 = 0.56
	BC	3 x EI / 5 = 0.60EI		0.60 / 1.35 = 0.44

01 M

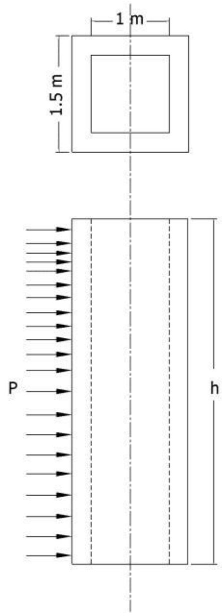
Joint	A	B	B	C
Member	AB	BA	BC	CB
Dist. Factor	1	0.56	0.44	1
FEM	-43.94	26.36	-52.08	52.08
Balancing	43.94	14.4	11.32	-52.08
C.O.		21.97	-26.04	
Balancing		2.28	1.79	
Final Moment	0	65.01	-65.01	0

02 M

Q.5 ***Attempt any two of the following:*** ***(16)***

a) ***A square chimney having external dimensions 1.5 m x 1.5m with wall thickness 250 mm is subjected to wind pressure 1.5kN/m^2. Find the maximum height of the chimney which can be allowed so that maximum stress in masonry is not to exceed 250 kN/m^2 compressive. Check whether masonry is safe if no tension is allowed. Consider weight of masonry 24 kN/m^3.***

Ans.



$p = 1.5$ kN/m^2; $\sigma_{max} = 250$ kN/m^2 Comp.; $\sigma = 24$ kN/m^3

Direct stress (σ_d) = σ x h = 24h -- 01 M
P = b x h x p = 1.5 x h x 1.5 = 2.25h
M = P x h/2 = 2.25h x h/2 = 1.125h^2 ------------------------------------ 01 M
I = (1.5^4 – 1^4) / 12 = 0.339 m^4 -- 01 M
y = 1.5 / 2 = 0.75 m.
Bending stress (σ_b) = M x y / I = 1.125h^2 x 0.75 / 0.339
= 2.49h^2 ------------------------------- 01 M
$\sigma_{max} = \sigma_d + \sigma_b$ -- 01 M
250 = 24h + 2.49h^2
h = 6.3 m. -- 01 M

$\sigma_{min} = \sigma_d - \sigma_b$ = 24 x 6.3 – 2.49 x 6.3^2 = 151.2 – 98.83
= 52.37 kN/m^2 (C) -------------------------------------- 01 M

Masonry is safe as no tension at base. ---------------------------------- 01 M

Q.5 b) ***Draw SFD and BMD for beam as shown in figure 4 using moment distribution method. (Figure 4).***

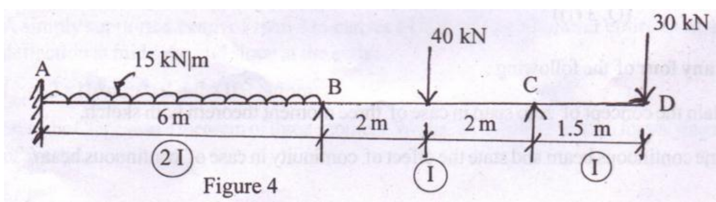


Figure 4

Ans.

M_{AB} = - 15 x 6^2 / 12 = - 45 kN-m M_{BA} = 15 x 6^2 / 12 = 45 kN-m
M_{BC} = - 40 x 4 / 8 = - 20 kN-m M_{CB} = 40 x 4 / 8 = 20 kN-m 01 M
M_{CD} = - 30 x 1.5 = - 45 kN-m

Joint	Member	Stiffness k	Σk	d = k/ Σk
B	BA	4 x 2EI/6 = 1.33EI	2.08EI	1.33 / 2.08 = 0.64
	BC	3 x EI / 4 = 0.75EI		0.75 / 2.08 = 0.36

01 M

Joint	A	B		C	
Member	AB	BA	BC	CB	CD
Dist. Factor	1	0.64	0.36	1	0
FEM	-45	45	-20	20	-45
Balancing		-16	-9	25	
C.O.	-8		12.05		
Balancing		-8	-4.5		
	-4				
Final Moment	-57	21	-21	45	-45

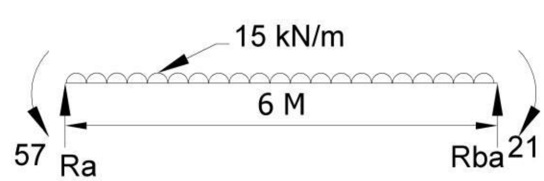
01 M

Reactions:

Span AB:

$\Sigma M_A = 0$

-57 + 21 – R_{ba} x 6 + 15 x 6 x 3 = 0



R_{ba} = 39 kN — 01 M

R_a = 15 x 6 – 39 = 51 kN.

Span BC:

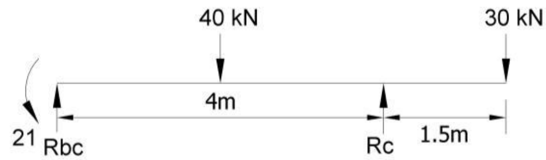
$\Sigma M_B = 0$

40 x 2 + 30 x 5.5 – 21 – R_c x 4 = 0

R_c = 56 kN

R_{bc} = 70 – 56 = 14 kN.

R_b = 39 + 14 = 53 kN.



Shear force calculations:

At D, just right = 0

At D just left = 30 kN — 01 M

At C, just right = 30 kN

At C, just left = 30 – 56 = - 26 kN

Under 40 kN, just right = - 26 kN

Under 40 kN, just left = - 26 + 40 = 14 kN

At B, just right = 14 kN

At B, just left = 14 -53 = - 39 kN

At A, just right = - 39 + 15 x 6 = 51 kN

Bending moment calculations:

At D = 0 — 01 M

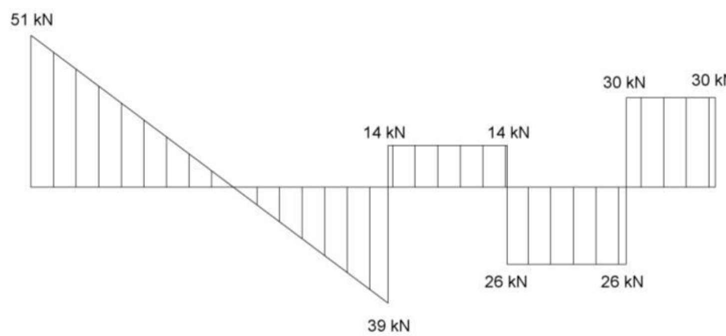
At B = - 30 x 1.5 = - 45 kN-m

Under 40 kN = - 30 x 3.5 + 56 x 2 = 7 kN-m

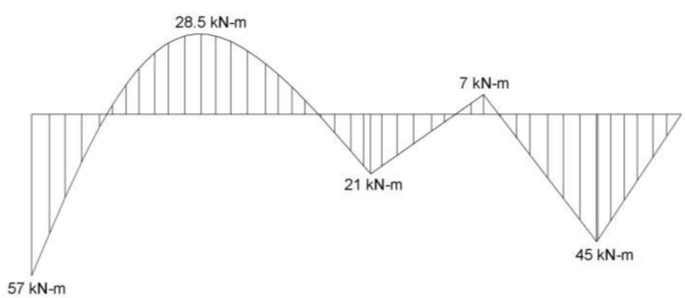
At B = MB = - 21 kN-m

At mid span AB = - 30 x 8.5 + 56 x 7 - 40 x 5 + 53 x 3 – 15 x 3 x 1.5 = 28.5 kN-m

At A = MA = - 57 kN-m.

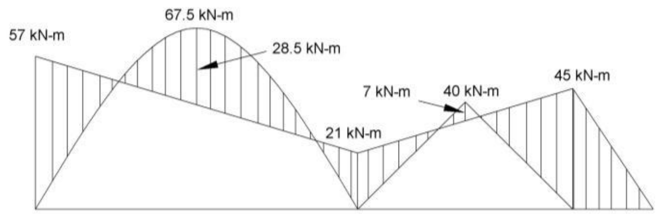


01 M



01 M

OR



Q.5 c) ***Determine the nature and magnitude of forces in the members AC, CE, DE, DB of frame as shown in figure 5. Also find support reactions. Use method of joints.***

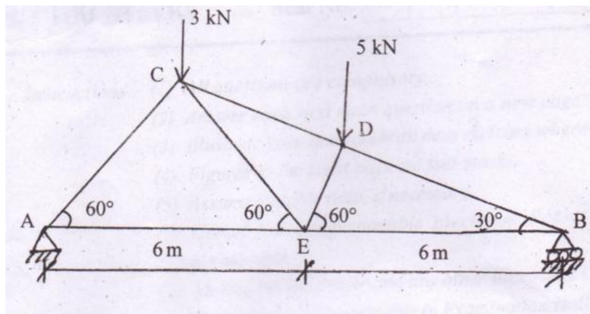
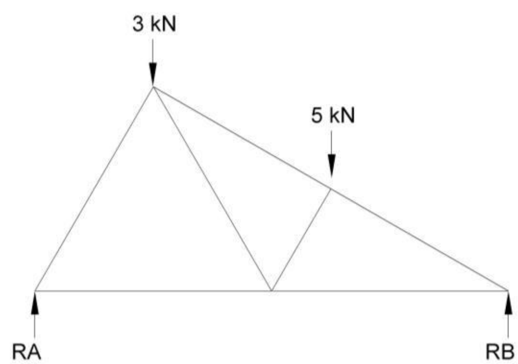


Figure 5

Ans.

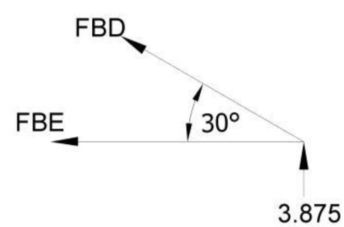


02 M for reactions

$R_B = 3 \times 3 + 5 \times (3 + 4.5) / 12 = 3.875$ kN

$R_A = 8 – 3.875 = 4.125$ kN

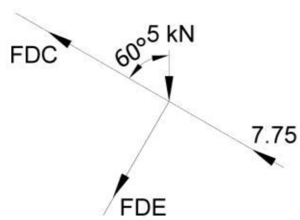
Joint B: -



$\Sigma F_y = FBD\sin30 + 3.875 = 0$

$F_{BD} = - 7.75$ kN   i.e. 7.75 kN (Comp)

Joint D: -



$\Sigma F_y = - 5\sin60 – FDE = 0$

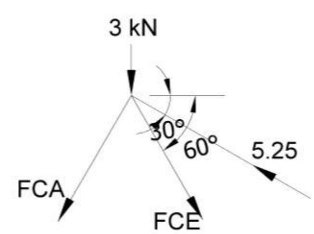
$F_{DE} = - 4.33$ kN   i.e. 4.33 kN (Comp)

$\Sigma F_x = - F_{DC} – 7.75 + 5\cos60 = 0$

$F_{DC} = - 5.25$ kN   i.e. 5.25 kN (Comp)

Three joints 02 M for each joint

Joint C: -



$\Sigma F_x = - F_{CA}\cos60 – 5.25\cos30 + F_{CE}\cos60 = 0$

$- 0.5F_{CA} – 4.547 + 0.5F_{CE} = 0$

$- 0.5F_{CA} + 0.5F_{CE} = 4.547$

$- F_{CA} + F_{CE} = 9.094$ (1)

$\Sigma F_y = - 3 – F_{CA}\sin60 – F_{CE}\sin60 + 5.25\sin30 = 0$

$- 3 – 0.866F_{CA} – 0.866F_{CE} + 2.625 = 0$

$-0.866F_{CA} - 0.866F_{CE} = 0.375$

$-F_{CA} - F_{CE} = 0.433$ (2)

Solving eq. 1 and eq.2 simultaneously

$F_{CA} = -4.76$ kN i.e. 4.76 kN (Comp)

$F_{CE} = 4.33$ kN (Tensile)

Q.6 ***Attempt any two of the following:*** ***(16)***

a) ***Find the slope and deflection at the centre of a simply supported beam as shown in figure 6 take EI =4000 kN.m^2. Use Macaulay's method.***

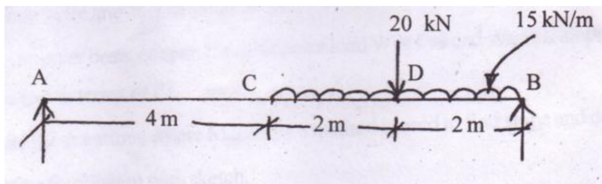
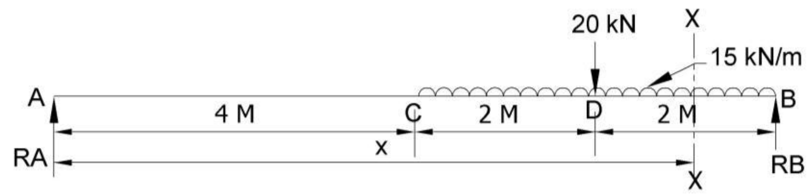


Figure 6

Ans.



$R_A = (20 \times 2 + 15 \times 4 \times 2) / 8 = 20$ kN

$R_B = 15 \times 4 + 20 - 20 = 60$ kN.

Taking section x-x at distance x from A between DB.

$Mx = 20 \times X - 20 \times (X - 6) - 15 \times (X - 4)^2 / 2$ -- 01 M

$EI(d^2y/dx^2) = -Mx = -20 \times X + 20 \times (X - 6) + 15 \times (X - 4)^2 / 2$

Integrating-

$EI(dy/dx) = -20 \times X^2/2 + 20 \times (X - 6)^2/2 + 15 \times (X - 4)^3 / 6 + C1$ (A) ------------------------- 01 M

Integrating-

$EIy = -20 \times X^3/6 + 20 \times (X - 6)^3/6 + 15 \times (X - 4)^4 / 24 + C_1 \times X + C_2$ (B) --------------------- 01 M

At X = 0; Y = 0 in equation B

$C_2 = 0$

At X = 8 m; Y = 0 in equation B

$0 = -20 \times 8^3/6 + 20 \times (8 - 6)^3/6 + 15 \times (8 - 4)^4 / 24 + C_1 \times 8 + C_2$

$C_1 = 190$ -- 01 M

Equations are-

$(dy/dx) = (1/EI)[-20 \times X^2/2 + 20 \times (X - 6)^2/2 + 15 \times (X - 4)^3 / 6 + 190]$ -----(1) ------------ 01 M

$y = (1/EI)[-20 \times X^3/6 + 20 \times (X - 6)^3/6 + 15 \times (X - 4)^4 / 24 + 190 \times X + 0]$ ------(2) ---------- 01 M

For slope at mid span, put X = 4 m in equation 1.

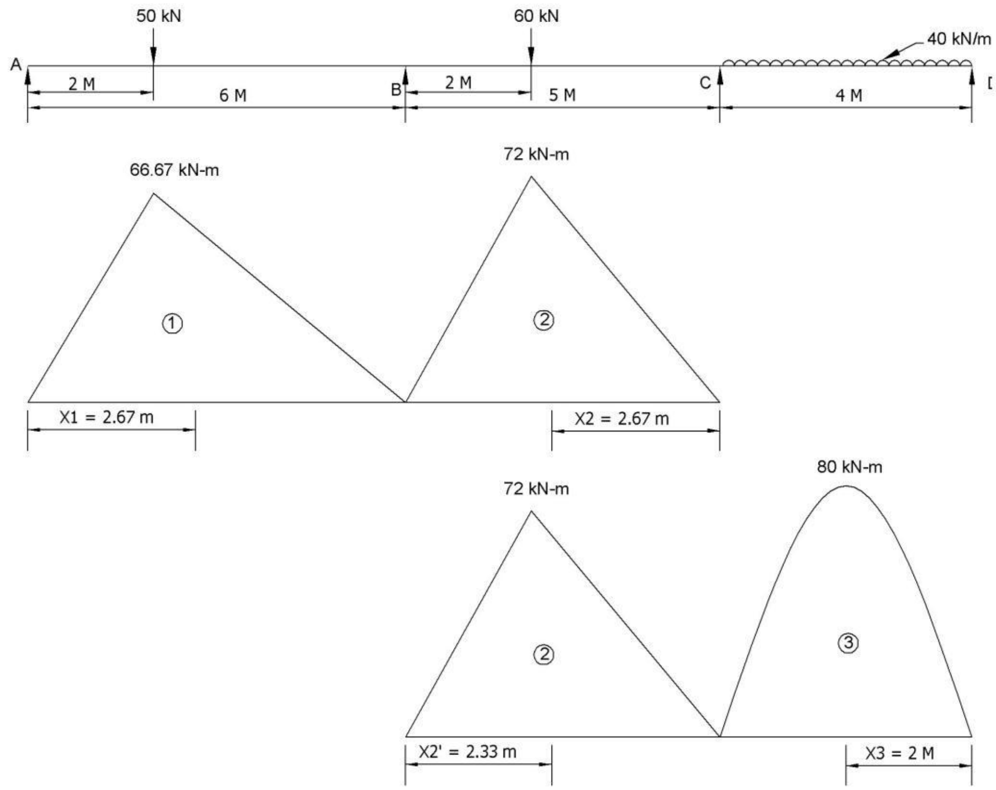
$(dy/dx)_C = (1/4000)[-20 \times 4^2/2 + 190] =$ **7.5×10^{-3} rad.** -- 01 M

For deflection at mid span, put X = 4 m in equation 2.

$y_C = (1/4000)[-20 \times 4^3/6 + 190 \times 4 + 0] =$ **0.1367 m i.e. 136.7 mm.** -------------------------- 01 M

Q.6 b) ***A continuous beam ABCD 15m long rests on supports A, B, C and D all at same level. Span AB = 6m, BC = 5 m and CD = 4 m. It carries two concentrated loads 50 kN and 60 kN at 2 m and 8 m from support A and UDL of 40 kN/m over span CD. Find the moments and reactions at the supports. Draw BMD using three moment theorem.***

Ans.



01 M

S.S.B.M. under 50 kN load = 50 x 2 x 4 / 6 = 66.67 kN-m

S.S.B.M. under 60 kN load = 60 x 2 x 3 / 5 = 72.0 kN-m — 01 M

S.S.B.M. at mid span of CD = 40 x 4^2 / 8 = 80.0 kN-m

A_1 = 0.5 x 6 x 66.67 = 200.00 X_1 = (6 + 2)/3 = 2.67 m

A_2 = 0.5 x 5 x 72 = 180 X_2 = (5 + 3)/3 = 2.67 m X_2' = (5 + 2)/3 = 2.33 m — 01 M

A_3 = 2 x 4 x 80 / 3 = 213.33 X_3 = 2 m

M_A = M_B = 0 End simple supports.

Applying theorem of three moment for span AB and BC-

M_A x L_1 + 2M_B(L_1 + L_2) + M_C x L_2 = - 6 [(A_1 x X_1 / L_1) + [(A_2 x X_2 / L_2)] — 01 M

0 + 2M_B(6 + 5) + M_C x 5 = - 6 [(200 x 2.67 / 6) + [(180 x 2.67 / 5)]

22M_B + 5M_C = - 6 (88.89 + 96)

= - 1109.33

M_B + 0.227M_C = - 50.424 --------- (1)

Applying theorem of three moment for span BC and CD-

M_B x L_2 + 2M_C(L_2 + L_3) + M_D x L_3 = - 6 [(A_2 x X_2' / L_2) + [(A_3 x X_3 / L_3)]

M_B x 5 + 2M_C(5 + 4) + 0 = - 6 [(180 x 2.33 / 5) + [(213.33 x 2.0 / 4)] — 01 M

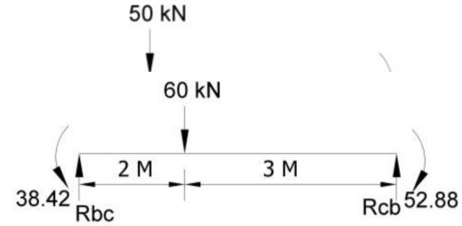
5M_B + 18M_C = - 6 (84 + 106.67)

= - 1144

M_B + 3.6M_C = - 228.8 --------- (2)

Solving equation 1 and 2 simultaneously.

M_C = - 52.88 kN-m and M_B = - 38.42 kN-m. — 01 M



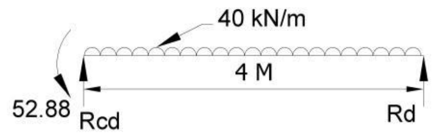
R_{ba} = (50 x 2 + 38.42)/6 = 23.07 kN

R_a = 50 – 23.07 = 26.93 kN

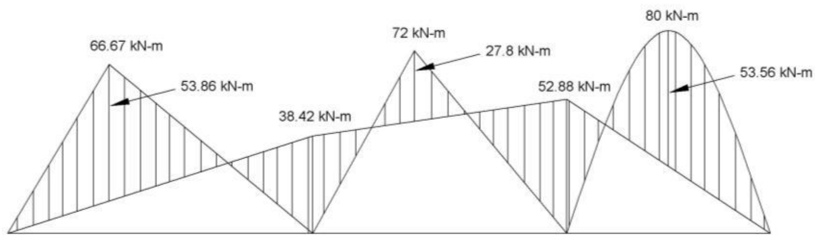
R_{cb} = (60 x 2 + 52.88 - 38.42)/5 = 26.892 kN — 01 M

R_{bc} = 60 – 26.892 = 33.108 kN

R_d = (40 x 4 x 2 – 42.88) = 66.78 kN
R_{cd} = 40 x 4 – 66.78 = 93.22 kN



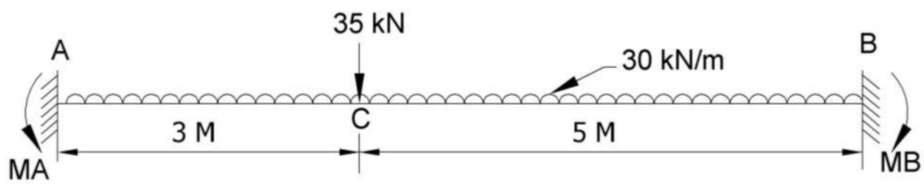
R_a = 26.93 kN; R_b = 23.07 + 33.108 = 56.178 kN
R_c = 26.892 + 93.22 = 120.112 kN; R_d = 66.78 kN



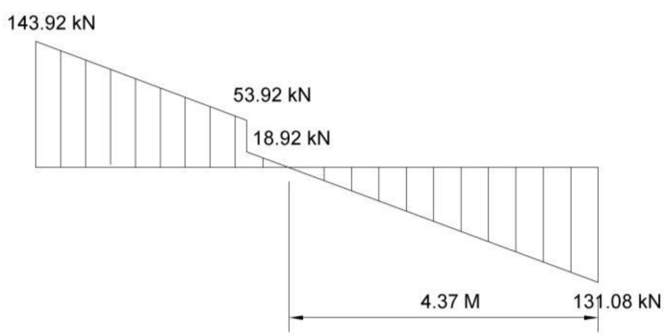
01 M

Q.6 c) ***A fixed beam 8m span is subjected to UDL of 30 kN/m over entire span along with point load of 35 kN acting 3m from left hand support. Calculate the net maximum sagging bending moment. Also draw SFD and BMD.***

Ans.

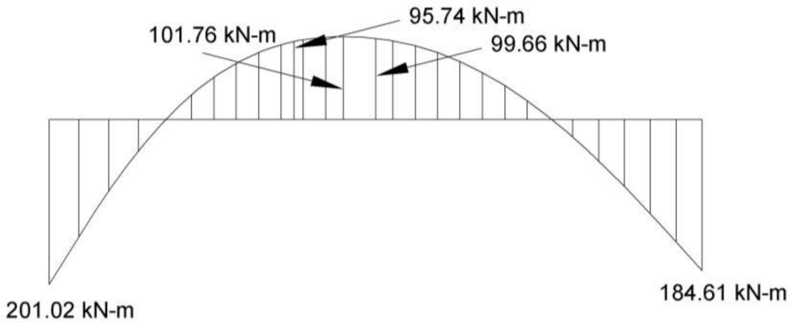


M_A = [(35 x 3 x 5^2) / 8^2] + [(30 x 8^2) / 12] = 201.02 kN-m — 01 M
M_B = [(35 x 5 x 3^2) / 8^2] + [(30 x 8^2) / 12] = 184.61 kN-m — 01 M
Reactions: -
R_B = [(35 x 3) + (30 x 8 x 4) + 184.61 – 201.02] / 8 = 131.08 kN — 01 M
R_A = 35 + 30 x 8 – 131.08 = 143.92 kN



01 M

Point of zero shear force, X = 131.08 / 30 = 4.37 m. — 01 M
Bending moment calculations: -
At A = - 201.02 kN-m.
At C = - 201.02 + 143.92 x 3 – 30 x 3^2 / 2 = 95.74 kN.m. — 01 M
At mid span = - 201.02 + 143.92 x 4 – 30 x 4^2 / 2 – 35 x 1 = 99.66 kN-m
Maximum bending moment at 4.37 m from B
= 131.08 x 4.37 – 184.61 – 30 x 4.37^2 / 2 = 101.76 kN-m — 01 M

			01 M