### Important Instructions to Examiners:

1. The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2. The model answer and the answer written by the candidate may vary but the examiner may try to assess the understanding level of the candidate.
3. The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills.)
4. While assessing figures, the examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and the model answer may vary. The examiner may give credit for any equivalent figure drawn.
5. Credits may be given step-wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.
6. In the case of some questions, credit may be given by judgement on part of the examiner regarding the candidate's understanding.
7. For programming language papers, credit may be given to any other program based on the equivalent concept.

### Question 1

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<th>Answer</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td>Attempt any TEN of the following:</td>
<td></td>
</tr>
<tr>
<td>a)</td>
<td>Ans</td>
<td>Find the point on the curve ( y = x^2 - 6x + 8 ) where the tangent is parallel to X-axis</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( y = x^2 - 6x + 8 )</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore \frac{dy}{dx} = 2x - 6 )</td>
<td>02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore ) tangent is parallel to X-axis</td>
<td>½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore \frac{dy}{dx} = 0 )</td>
<td>½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore 2x - 6 = 0 )</td>
<td>½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore x = 3 )</td>
<td>½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( y = 3^2 - 6(3) + 8 = -1 )</td>
<td>½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore ) Point is (3, -1)</td>
<td>½</td>
</tr>
<tr>
<td>b)</td>
<td>Ans</td>
<td>Find the radius of curvature of the curve ( xy = c ) at point ((c, c))</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( xy = c )</td>
<td>02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x \frac{dy}{dx} + y = 0 )</td>
<td></td>
</tr>
</tbody>
</table>
### Q. No. 1

#### b)

\[ \frac{dy}{dx} = -\frac{y}{x} \]

\[ \frac{d^2y}{dx^2} = -\left( x \frac{dy}{dx} - y \right) \]

At \((c,c)\),

\[ \frac{dy}{dx} = -\frac{c}{c} = -1 \]

\[ \frac{d^2y}{dx^2} = -\left( \frac{c(-1) - c}{c^2} \right) = \frac{2}{c} \]

Radius of curvature:

\[ \left( 1 + \left( \frac{dy}{dx} \right)^2 \right)^{\frac{1}{2}} \]

\[ \frac{d^2y}{dx^2} \]

\[ = \frac{c}{2} \left( 2\sqrt{2} \right) = \sqrt{2} \cdot c \]

---

#### c)

Evaluate \(\int \frac{1}{\sin^{-1} x \sqrt{1-x^2}} dx\)

**Ans**

\[ \int \frac{1}{\sin^{-1} x \sqrt{1-x^2}} dx \]

Put \(\sin^{-1} x = t\)

\[ \frac{1}{\sqrt{1-x^2}} dx = dt \]

\[ = \int \frac{1}{t} dt \]

\[ = \log t + c \]

\[ = \log (\sin^{-1} x) + c \]

---

#### d)

Evaluate \(\int \frac{1}{\sqrt{(2-3x)^3}} dx\)

**Ans**

\[ \int \frac{1}{\sqrt{(2-3x)^3}} dx \]
### SUMMER – 17 EXAMINATION

**Subject Code:** 17301

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<tr>
<th>Q. No.</th>
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<th>Answer</th>
<th>Marking Scheme</th>
</tr>
</thead>
</table>
| 1      | d)        | \[
\int \frac{1}{(2 - 3x)^{\frac{3}{4}}} \, dx
\]
\[
= \int (2 - 3x)^{-\frac{3}{4}} \, dx
\]
\[
= \frac{1}{-\frac{3}{4} + 1} \cdot \frac{-1}{3} = \frac{1}{3} + c
\]
\[
= -\frac{4}{3} (2 - 3x)^{\frac{1}{3}} + c
\] | \(\frac{1}{2}\) |

| e) | Evaluate \(\int \tan^{-1} x \, dx\) | \[
\int \tan^{-1} x \cdot 1 \, dx
\]
\[
= \tan^{-1} x \int 1 \, dx - \int \left( \int 1 \, dx \cdot \frac{d}{dx} \left( \tan^{-1} x \right) \right) \, dx
\]
\[
= \tan^{-1} x \cdot x - \int \frac{x}{1 + x^2} \, dx
\]
\[
= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1 + x^2} \, dx
\]
\[
= x \tan^{-1} x - \frac{1}{2} \log (1 + x^2) + c
\] | 02 |

| f) | Evaluate \(\int_0^{\pi/2} \sin^3 x \, dx\) | \[
\int_0^{\pi/2} \sin^3 x \, dx
\]
\[
= \int_0^{\pi/2} \sin x \cdot \sin x \cdot \sin x \, dx
\]
\[
= \int_0^{\pi/2} \sin x \cdot \sin x \cdot \cos x \, dx
\]
\[
= \int_0^{\pi/2} (1 - \cos^2 x) \sin x \, dx
\] | 02 |

Put \(\cos x = t\) when \(x = 0 , t = 1\)

\[
\therefore -\sin x \, dx = dt
\] when \(x = \frac{\pi}{2} , t = 0\) | 02 |
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>f)</td>
<td>[ \int_0^1 (1 - t^2) , dt ] [ \int_0^1 (-1 + t^2) , dt ] [ -t + \frac{t^3}{3} \Big</td>
<td>_0^1 ] [ = (0) - \left(-1 + \frac{1}{3}\right) ] [ = \frac{2}{3} ]</td>
</tr>
<tr>
<td></td>
<td>OR</td>
<td>[ \int_0^{\pi/2} \sin^3 x , dx ] [ \frac{\pi}{2} ] [ = \int_0^{\pi/2} \frac{3\sin x - \sin 3x}{4} , dx ] [ = \frac{1}{4} \left[ 3(-\cos x) + \frac{\cos 3x}{3} \right]^{\pi/2}_0 ] [ = \frac{1}{4} \left[ 0 + 0 + 3 - \frac{1}{3} \right] ] [ = \frac{2}{3} ]</td>
<td>( \frac{1}{2} ) ( \frac{1}{2} ) ( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td>g)</td>
<td>Find the area of the region bounded by ( x^2 = 16y ), ( y = 1 ), ( y = 4 ) and Y-axis in first quadrant</td>
<td>02</td>
</tr>
<tr>
<td>Ans</td>
<td></td>
<td>[ A = \int_a^b x , dy ] [ = \int_1^4 4\sqrt{y} , dy ] [ = 4 \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 ]</td>
<td>( \frac{1}{2} ) ( \frac{1}{2} )</td>
</tr>
</tbody>
</table>
### SUMMER – 17 EXAMINATION

**Model Answer**

<table>
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<tr>
<td>1</td>
<td>g)</td>
<td>$\frac{8}{3} \left( \frac{2}{3} - 1^2 \right)$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>h)</td>
<td>Determine the order and degree of $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = my$</td>
<td>02</td>
</tr>
<tr>
<td></td>
<td>Ans</td>
<td>Order=2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Degree=1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>i)</td>
<td>Form the differential equation if $y = 4(x - A)^2$. Where $A$ is arbitrary constant</td>
<td>02</td>
</tr>
<tr>
<td></td>
<td>Ans</td>
<td>$y = 4(x - A)^2$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\therefore \frac{dy}{dx} = 8(x - A)$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y = 4 \left( \frac{1}{8} \frac{dy}{dx} \right)^2$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y = 4 \frac{1}{64} \left( \frac{dy}{dx} \right)^2$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\therefore \left( \frac{dy}{dx} \right)^2 = 16y$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>j)</td>
<td>A fair die is rolled. What is the probability that the number on the die is a prime number</td>
<td>02</td>
</tr>
<tr>
<td></td>
<td>Ans</td>
<td>$S = {1, 2, 3, 4, 5, 6}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\therefore n(s) = 6$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A = {2, 3, 5}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\therefore n(A) = 3$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p(A) = \frac{n(A)}{n(s)} = \frac{3}{6}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= \frac{1}{2}$ or 0.5</td>
<td>1</td>
</tr>
<tr>
<td>Q. No.</td>
<td>Sub Q. N.</td>
<td>Answer</td>
<td></td>
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<tr>
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<tr>
<td>1 k)</td>
<td>Ans</td>
<td>From 4 men and 2 women, 3 persons are chosen at random to form a committee. Find the probability that the committee consists of at least one person of either sex.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n(S) = \binom{6}{3} = 20 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n(A) = \binom{4}{1} \times \binom{2}{2} + \binom{4}{2} \times \binom{2}{1} = 16 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( p(A) = \frac{16}{20} = \frac{4}{5} ) or 0.8</td>
<td></td>
</tr>
<tr>
<td>1 l)</td>
<td>Ans</td>
<td>A person fires 10 shorts at target. The probability that any shot will hit the target is ( \frac{3}{5} ). Find the probability that the target is hit exactly 5 times.</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Given : ( n = 10, p = \frac{3}{5}, q = 1 - p = 1 - \frac{3}{5} = \frac{2}{5} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( p(r) = \binom{n}{r} p^r q^{n-r} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( p(5) = \binom{10}{5} \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5 = 0.2007 )</td>
<td></td>
</tr>
<tr>
<td>1 m)</td>
<td>Ans</td>
<td>Evaluate ( \int \frac{1}{\sqrt{9-4x^2}} , dx )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = \int \frac{1}{\sqrt{3^2-(2x)^2}} , dx ) OR ( = \int \frac{1}{\sqrt{\frac{9}{4}-x^2}} , dx )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = \frac{1}{2} \int \frac{1}{\sqrt{\frac{3^2}{4^2}-x^2}} , dx )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = \sin^{-1} \left( \frac{2x}{3} \right) \frac{1}{2} + c ) OR ( = \frac{1}{2} \sin^{-1} \left( \frac{x}{3/2} \right) + c = \frac{1}{2} \sin^{-1} \left( \frac{2x}{3} \right) + c )</td>
<td></td>
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### Question 1

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<tbody>
<tr>
<td>1</td>
<td>n)</td>
<td>Evaluate $\int_1^e \frac{1}{x} \log x , dx$</td>
<td></td>
</tr>
</tbody>
</table>

**Ans.**

$\int_1^e \frac{1}{x} \log x \, dx$

Put $\log x = t$

- when $x = 1$, $t = \log 1 = 0$
- when $x = e$, $t = \log e = 1$

$\frac{1}{x} \, dx = dt$

$\int_1^e t \, dt$

$\left[ \frac{t^2}{2} \right]_0^1$

$= \frac{1}{2}$

---

### Question 2

**Attempt any FOUR of the following:**

**a)** Evaluate $\int \tan^2 x \, dx$

Considering index as 3

$\therefore \int \tan^2 x \, dx$

$= \int \tan^2 x \tan x \, dx$

$= \int (\sec^2 x - 1) \tan x \, dx$

$= \int (\tan x \sec^2 x - \tan x) \, dx$

$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx$

In first integral, put $\tan x = t$

$\therefore \sec^2 x \, dx = dt$

$= \int t \, dt - \log (\sec x) + c$

$= \frac{t^2}{2} - \log (\sec x) + c$

$= \frac{\tan^2 x}{2} - \log (\sec x) + c$

**Note:** If student attempted to solve the problem assuming any index value then consider it and reward appropriate marks to it.

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### Model Answer

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<tr>
<td>2</td>
<td>b)</td>
<td>Evaluate ( \int \frac{\log x}{x(2 + \log x)(3 + \log x)} , dx )</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \int \frac{\log x}{x(2 + \log x)(3 + \log x)} , dx )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Put ( \log x = t )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore \frac{1}{x} , dx = dt )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \int \frac{t}{(2+t)(3+t)} , dt )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>consider ( \frac{t}{(2+t)(3+t)} = \frac{A}{2+t} + \frac{B}{3+t} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore t = A(3+t) + B(2+t) )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Put ( t = -2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( A = -2 )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Put ( t = -3 )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( B = 3 )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore \frac{t}{(2+t)(3+t)} = -\frac{2}{2+t} + \frac{3}{3+t} )</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
<td>Find the equations of the tangent and normal to the ellipse ( 2x^2 + 3y^2 = 5 ) which is perpendicular to the line ( 3x + 2y + 7 = 0 )</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Ans</td>
<td>Slope of line ( 3x + 2y + 7 = 0 ) is</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( m_1 = -\frac{3}{2} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore 2x^2 + 3y^2 = 5 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore 4(x) + 6y \frac{dy}{dx} = 0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \therefore \frac{dy}{dx} = \frac{-4x}{6y} = -\frac{2x}{3y} )</td>
<td></td>
</tr>
</tbody>
</table>
### Q. No. 2 c)  

: slope of tangent \( m_2 = \frac{-2x}{3y} \)

Line and tangent are perpendicular

\[ m_1 m_2 = -1 \]

\[ \frac{-3}{2} \cdot \frac{-2x}{3y} = -1 \]

\[ x = -y \quad \therefore y = -x \]

\[ 2x^2 + 3(-x)^2 = 5 \]

\[ 2x^2 + 3x^2 = 5 \]

\[ x^2 = 1 \]

\[ x = \pm 1 \]

If \( x = 1 \) \( y = -1 \) \( \therefore \) point is \((1, -1)\)

If \( x = -1 \) \( y = 1 \) \( \therefore \) point is \((-1, 1)\)

Equation of tangent at \((1, -1)\) is

\[ y + 1 = \frac{2}{3}(x - 1) \]

\[ 2x - 3y - 5 = 0 \]

Equation of tangent at \((-1, 1)\) is

\[ y - 1 = \frac{2}{3}(x + 1) \]

\[ 2x - 3y + 5 = 0 \]

Equation of normal at \((1, -1)\)

\[ y + 1 = \frac{-3}{2}(x - 1) \]

\[ 3x + 2y - 1 = 0 \]

Equation of normal at \((-1, 1)\)

\[ y - 1 = \frac{-3}{2}(x + 1) \]

\[ 3x + 2y + 1 = 0 \]

---

d) Find the radius of curvature for the curve \( x = a \cos^3 \theta \), \( y = a \sin^3 \theta \) at \( \theta = \frac{\pi}{4} \)

Ans

\[
\begin{align*}
\frac{dx}{d\theta} &= -3a \cos^2 \theta \sin \theta \\
\frac{dy}{d\theta} &= 3a \sin^2 \theta \cos \theta
\end{align*}
\]

\[ \frac{1}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}} \]

04

\[ \frac{1}{2} + \frac{1}{2} \]
<table>
<thead>
<tr>
<th>Q. No.</th>
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</tr>
</thead>
</table>
| 2     | d)       | \[
\frac{dy}{dx} = \frac{d\theta}{d\theta} = \frac{3a \sin^2 \theta \cos \theta}{-3a \cos^2 \theta \sin \theta}
\]
\[
\frac{dy}{dx} = -\tan \theta
\]
\[
\frac{d^2y}{dx^2} = -\sec^2 \theta \frac{d\theta}{dx}
\]
\[
= -\sec^2 \theta \frac{1}{dx} \frac{d\theta}{d\theta}
\]
\[
= -\sec^2 \theta \frac{1}{-3a \cos^2 \theta \sin \theta}
\]
\[\text{at } \theta = \frac{\pi}{4}\]
\[
\frac{dy}{dx} = -\tan \left(\frac{\pi}{4}\right) = -1
\]
\[
\frac{d^2y}{dx^2} = -\sec^2 \left(\frac{\pi}{4}\right) \frac{1}{-3a \cos^2 \left(\frac{\pi}{4}\right) \sin \left(\frac{\pi}{4}\right)}
\]
\[= -\left(\frac{\sqrt{2}}{2}\right)^3 \frac{1}{-3a \left(\frac{1}{\sqrt{2}}\right)^2 \left(\frac{1}{\sqrt{2}}\right)}
\]
\[= \frac{2}{3a \sqrt{2}}
\]
\[= \frac{4\sqrt{2}}{3a}
\]
\[\text{Radius of curvature } = \frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}^{\frac{3}{2}}
\]
\[= \frac{\left(1 + (-1)^2\right)^{\frac{3}{2}}}{\frac{4\sqrt{2}}{3a}}
\]
\[= \frac{3a}{2} \text{ or } (1.5)a
\]
### Q. No. 2

**Ans e)** A bullet is fired into a mud tank and penetrates \( (120t - 3600t^2) \) meters in ‘t’ seconds after impact. Calculate maximum depth of penetration.

\[
y = 120t - 3600t^2
\]

\[
\frac{dy}{dt} = 120 - 7200t
\]

\[
\frac{d^2y}{dt^2} = -7200 < 0 \quad \therefore \text{Depth is maximum.}
\]

Let \( \frac{dy}{dt} = 0 \)

\[
120 - 7200t = 0
\]

\[
\therefore t = \frac{1}{60}
\]

\[
\therefore \text{Maximum depth } y = 120 \left( \frac{1}{60} \right) - 3600 \left( \frac{1}{60} \right)^2
\]

\[
= 1 \text{ meter}
\]

**Ans f)** Evaluate \( \int \frac{5x-4}{x^2-8x+12} \, dx \)

\[
\int \frac{5x-4}{x^2-8x+12} \, dx
\]

Consider \( \frac{5x-4}{x^2-8x+12} = \frac{5x-4}{(x-6)(x-2)} \)

\[
\frac{5x-4}{(x-6)(x-2)} = \frac{A}{x-6} + \frac{B}{x-2}
\]

\[
\therefore 5x-4 = A(x-2) + B(x-6)
\]

Put \( x = 6 \)

\[
\therefore A = \frac{26}{4} = \frac{13}{2}
\]

Put \( x = 2 \)

\[
\therefore B = \frac{-6}{4} = \frac{-3}{2}
\]

\[
\frac{5x-4}{x^2-8x+12} = \frac{13}{2} \frac{1}{x-6} + \frac{-3}{2} \frac{1}{x-2}
\]

\[
\therefore \int \frac{5x-4}{x^2-8x+12} \, dx = \int \left( \frac{13}{2} \frac{1}{x-6} + \frac{-3}{2} \frac{1}{x-2} \right) \, dx
\]
### SUMMER – 17 EXAMINATION

**Model Answer**

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<tr>
<td>2</td>
<td>f)</td>
<td>( I = \frac{13}{2} \log(x - 6) - \frac{3}{2} \log(x - 2) + c )</td>
</tr>
</tbody>
</table>

---

**Attempt any FOUR of the following:**

a) Evaluate \( \int_{a}^{b} \frac{1}{4x^2 + 12x + 13} \, dx \)

Ans

\[
\frac{1}{4} \int_{a}^{b} \frac{1}{\sqrt{2} \left( x^2 + 3x + \frac{13}{4} \right)} \, dx
\]

Third term = \( \frac{(3x)^2}{4(x)^2} = \frac{9}{4} \)

\[
\frac{1}{4} \int_{a}^{b} \frac{1}{\sqrt{2} \left( x + \frac{3}{2} \right)^2 + 1^2} \, dx
\]

\[
= \left[ \frac{1}{4} \tan^{-1} \left( x + \frac{3}{2} \right) \right]_{a}^{b}
\]

\[
= \frac{1}{4} \tan^{-1} \left( \frac{1}{2} + \frac{3}{2} \right) - \frac{1}{4} \tan^{-1} \left( -\frac{3}{2} + \frac{3}{2} \right)
\]

\[
= \frac{1}{4} \tan^{-1} (2)
\]

---

b) Evaluate \( \int_{a}^{b} \frac{1}{\sqrt{x + 3 \cot x}} \, dx \)

Ans

\[
\int_{a}^{b} \frac{1}{\sqrt{x + 3 \cot x}} \, dx
\]
### Model Answer

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</table>
| 3     | b)        | $I = \int_{\pi/6}^{\pi/2} \frac{1}{1 + \sqrt{\cot x}} \, dx$  

$= \int_{\pi/6}^{\pi/2} \frac{1}{\cos x} \, dx$  

$= \int_{\pi/6}^{\pi/2} \frac{1}{\sqrt{\sin x}} \, dx$  

$I = \int_{\pi/6}^{\pi/2} \frac{\sqrt{\sin x}}{\sin x + \sqrt{\cos x}} \, dx$  

$\cdots (1)$  

$I = \int_{\pi/6}^{\pi/2} \sqrt{\sin \left( \frac{\pi}{3} + \frac{\pi}{6} - x \right)} \, dx$  

$\sqrt{\sin \left( \frac{\pi}{3} + \frac{\pi}{6} - x \right) + \cos \left( \frac{\pi}{3} + \frac{\pi}{6} - x \right)}$  

$\therefore I = \int_{\pi/6}^{\pi/2} \sqrt{\sin \left( \frac{\pi}{2} - x \right)} \, dx$  

$\therefore I = \int_{\pi/6}^{\pi/2} \frac{\sqrt{\cos x}}{\cos x + \sqrt{\sin x}} \, dx$  

$\cdots (2)$  

add (1) and (2)  

$I + I = \int_{\pi/6}^{\pi/2} \frac{\sqrt{\sin x}}{\sin x + \sqrt{\cos x}} \, dx + \int_{\pi/6}^{\pi/2} \frac{\sqrt{\cos x}}{\cos x + \sqrt{\sin x}} \, dx$  

$2I = \int_{\pi/6}^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sin x + \sqrt{\cos x}} \, dx$  

$2I = \int_{\pi/6}^{\pi/2} 1 \, dx$  

$2I = \left[ x \right]_{\pi/6}^{\pi/2}$  

$2I = \frac{\pi}{3} - \frac{\pi}{6}$  

$I = \frac{\pi}{12}$ |

**Marking Scheme**  

<table>
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<tbody>
<tr>
<td>3</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>b)</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$I = \int_{\pi/6}^{\pi/2} \frac{1}{1 + \sqrt{\cot x}} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\int_{\pi/6}^{\pi/2} \frac{1}{\cos x} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\int_{\pi/6}^{\pi/2} \frac{1}{\sqrt{\sin x}} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\int_{\pi/6}^{\pi/2} \frac{\sqrt{\sin x}}{\sin x + \sqrt{\cos x}} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\cdots (1)$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\int_{\pi/6}^{\pi/2} \sqrt{\sin \left( \frac{\pi}{3} + \frac{\pi}{6} - x \right)} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\sqrt{\sin \left( \frac{\pi}{3} + \frac{\pi}{6} - x \right) + \cos \left( \frac{\pi}{3} + \frac{\pi}{6} - x \right)}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\therefore I = \int_{\pi/6}^{\pi/2} \sqrt{\sin \left( \frac{\pi}{2} - x \right)} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\therefore I = \int_{\pi/6}^{\pi/2} \frac{\sqrt{\cos x}}{\cos x + \sqrt{\sin x}} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\cdots (2)$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\int_{\pi/6}^{\pi/2} \frac{\sqrt{\sin x}}{\sin x + \sqrt{\cos x}} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\int_{\pi/6}^{\pi/2} \frac{\sqrt{\cos x}}{\cos x + \sqrt{\sin x}} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$2I = \int_{\pi/6}^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sin x + \sqrt{\cos x}} , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$2I = \int_{\pi/6}^{\pi/2} 1 , dx$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$2I = \left[ x \right]_{\pi/6}^{\pi/2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$2I = \frac{\pi}{3} - \frac{\pi}{6}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$I = \frac{\pi}{12}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Q. No.</td>
<td>Sub Q. N.</td>
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<td>--------</td>
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<tr>
<td>3</td>
<td>c)</td>
</tr>
<tr>
<td>d)</td>
<td>Ans</td>
</tr>
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</table>
### Q. No. 3 d)

\[ v + x \frac{dv}{dx} = \frac{x^2 (1 + v + v^2)}{x^2} \]
\[ v + x \frac{dv}{dx} = 1 + v + v^2 \]
\[ x \frac{dv}{dx} = 1 + v^2 \]
\[ x dv = (1 + v^2) dx \]
\[ \frac{1}{1 + v^2} dv = \frac{1}{x} dx \]
\[ \int \frac{1}{1 + v^2} dv = \int \frac{1}{x} dx \]
\[ \tan^{-1}(v) = \log x + c \]
\[ \tan^{-1}\left(\frac{y}{x}\right) = \log x + c \]

### e)

Solve \( \cos^2(x - 2y) = 1 - 2 \frac{dy}{dx} \)

\( \cos^2(x - 2y) = 1 - 2 \frac{dy}{dx} \)

Put \( x - 2y = v \)
\[ 1 - 2 \frac{dy}{dx} = \frac{dv}{dx} \]
\[ \therefore \cos^2 v = \frac{dv}{dx} \]
\[ dx = \frac{1}{\cos^2 v} dv \]
\[ \therefore \] solution is
\[ \int dx = \int \sec^2 v dv \]
\[ x = \tan v + c \]
\[ x = \tan (x - 2y) + c \]

### f)

Solve \( (1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1}x} \)

\[ \therefore \frac{dy}{dx} + \frac{1}{1 + x^2} y = e^{\tan^{-1}x} \]

Comparing with \( \frac{dy}{dx} + Py = Q \)
### Question 3 f) 
\[ P = \frac{1}{1 + x^2}, \quad Q = \frac{e^{\tan^{-1} x}}{1 + x^2} \]

Integrating factor: 
\[ e^{\int P \, dx} = e^{\int \frac{1}{1 + x^2} \, dx} = e^{\tan^{-1} x} \]

\[ y.IF = \int Q.IF \, dx \]

\[ y.e^{\tan^{-1} x} = \int \left( e^{\tan^{-1} x} \right) \frac{1}{1 + x^2} \, dx \]

\[ ye^{\tan^{-1} x} = \int \left( e^{\tan^{-1} x} \right)^2 \, dx \]

Put \( \tan^{-1} x = t \)  
OR  
Put \( e^{\tan^{-1} x} = t \)

\[ \frac{1}{1 + x^2} \, dx = dt \]

\[ ye^{\tan^{-1} x} = \int \left( e^t \right)^2 \, dt = \int e^{2t} \, dt \]

\[ ye^{\tan^{-1} x} = \frac{e^{2t}}{2} + c \]

\[ ye^{\tan^{-1} x} = \frac{e^{2\tan^{-1} x}}{2} + c \]

### Question 4 a) 
Evaluate \( \int \frac{(10 - x)^2}{x^2 + (10 - x)^2} \, dx \)

Let \( I = \int \frac{(10 - x)^2}{x^2 + (10 - x)^2} \, dx \)  (1)

\[ I = \int \frac{(10 - x)^2}{x^2 + (10 - (7 + 3 - x))^2} \, dx \]

\[ I = \int \frac{(10 - x)^2}{(7 + 3 - x)^2 + (10 - (7 + 3 - x))^2} \, dx \]  (2)

Adding (1) and (2)

\[ I + I = \int \frac{(10 - x)^2}{x^2 + (10 - x)^2} + \int \frac{x^2}{(10 - x)^2 + x^2} \, dx \]

---

**Marking Scheme**

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<tbody>
<tr>
<td>3 f)</td>
<td></td>
<td></td>
<td>( \frac{1}{2} )</td>
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<tr>
<td>4 a)</td>
<td></td>
<td></td>
<td>16</td>
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<tbody>
<tr>
<td>4</td>
<td>a)</td>
<td>(2I = \int \frac{7(10-x)^2 + x^2}{x^2 + (10-x)^2} , dx)</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2I = \int 1 , dx)</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2I = [x]_3^7)</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2I = 7 - 3)</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(I = \frac{4}{2})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(I = 2)</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>b)</td>
<td>Evaluate (\int x \sin^{-1} x , dx)</td>
<td>04</td>
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</table>

**Ans**

\[
\int x \sin^{-1} x \, dx
= \sin^{-1} x \int x \, dx - \int \left[ \int_0^1 x \, d\left(\sin^{-1} x\right) \right] \, dx
= \sin^{-1} x \cdot \frac{x^2}{2} - \int_0^1 \left( \frac{x^2}{2} \cdot \frac{1}{\sqrt{1-x^2}} \right) \, dx
= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int_0^1 \left( 1 - x^2 - 1 \right) \, dx
= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int_0^1 \left( \frac{1 - x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right) \, dx
= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left[ \left( \frac{1 - x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right) \right]_0^1
= \left[ \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left( \frac{x}{\sqrt{1-x^2}} + \frac{1^2}{2} \sin^{-1} (x) - \sin^{-1} x \right) \right]_0^1
= \left[ \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} - \frac{1}{4} \sin^{-1} (x) \right]_0^1
= \left[ \frac{1^2 \sin^{-1} (1)}{2} + 0 - \frac{1}{4} \sin^{-1} (1) \right] - 0
= \frac{\pi}{2} - \frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8}
\]
### Model Answer

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</table>
| 4 | c) | Find the area of the region in the first quadrant enclosed by the X-axis the line \( y = x \) and the circle \( x^2 + y^2 = 8 \)  
\[ \therefore y = x, \quad x^2 + y^2 = 8 \]  
\[ \therefore x^2 + x^2 = 8 \]  
\[ 2x^2 = 8 \]  
\[ x^2 = 4 \]  
\[ \therefore x = \pm 2 \quad \text{In first quadrant} \]  
\( x = 0 \) to \( x = 2 \)  
Due to symmetry about the line \( y=x \) the region in first quadrant of the circle is divided into two equal parts and hence can be integrated as follows:  
\[ A = \int_{a}^{b} (y_2 - y_1) \]  
\[ \therefore A = \int_{0}^{2} \left( \sqrt{8} - x - x \right) dx \]  
\[ \therefore A = \left[ \frac{x}{2} \sqrt{8} - x^2 + \frac{\sqrt{8}}{2} \sin^{-1}\left(\frac{x}{\sqrt{8}}\right) - \frac{x^2}{2} \right]_0 \]  
\[ \therefore A = \left[ \frac{2}{2} \sqrt{8} - (2)^2 + \frac{\sqrt{8}}{2} \sin^{-1}\left(\frac{2}{\sqrt{8}}\right) - \left(\frac{2}{2}\right)^2 \right] - 0 \]  
\[ \therefore A = 2 + 4 \left(\frac{\pi}{4}\right) - 2 \]  
\[ = \pi \]  
**OR**  
\[ \therefore y = x, \quad x^2 + y^2 = 8 \]  
\[ y^2 = 4 \]  
\[ \therefore y = \pm 2 \quad \text{In first quadrant} \]  
\( y = 0 \) to \( y = 2 \)  
\[ A = \int_{a}^{b} (x_2 - x_1) dy \]  
\[ \therefore A = \int_{0}^{2} \left( \sqrt{8} - y^2 - y \right) dy \]    | 04 |

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### SUMMER – 17 EXAMINATION

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</table>
| 4      | c)        | \[ A = \left[ \frac{y}{2} \sqrt{8} - y^2 + \left( \frac{\sqrt{8}}{2} \right)^2 \sin^{-1} \left( \frac{y}{\sqrt{8}} \right) - \frac{y^2}{2} \right]_0^2 \]  
\[ A = \frac{2}{2} \sqrt{8} - (2)^2 + \left( \frac{\sqrt{8}}{2} \right)^2 \sin^{-1} \left( \frac{2}{\sqrt{8}} \right) - \frac{(2)^2}{2} - 0 \]  
\[ A = 2 + 4 \left( \frac{\pi}{4} \right) - 2 \]  
\[ = \pi \] | 1 |

#### OR

Find the area of the region in the first quadrant enclosed by the X-axis the line \( y = x \) and the circle \( x^2 + y^2 = 8 \)

\[ y = x, \ x^2 + y^2 = 8 \]

\[ x^2 + x^2 = 8 \]

\[ 2x^2 = 8 \]

\[ x^2 = 4 \]

\[ x = \pm 2 \]

\[ \therefore \text{In first quadrant point of intersection is (2, 2)} \]

Let \( BM \perp X-axis \) \[ \therefore \text{Required area} = A(\text{region OBMO}) + A(\text{region BAMB}) \]

\[ A(\text{region OBMO}) = \int_0^2 y \ dx \]

\[ = \int_0^2 x \ dx \]

\[ = \left[ \frac{x^2}{2} \right]_0^2 = 2 \] | \( \frac{1}{2} \) |
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<tr>
<td>4</td>
<td>c)</td>
<td>( A(\text{region } BAMB) = \int_{0}^{\sqrt{2}} y , dx )</td>
</tr>
</tbody>
</table>

\[
= \int_{0}^{\sqrt{2}} \sqrt{8-x^2} \, dx \\
= \left[ \frac{x}{2} \sqrt{(\sqrt{8})^2-x^2} + \left( \frac{\sqrt{8}}{2} \right)^2 \sin^{-1} \left( \frac{x}{\sqrt{8}} \right) \right]_{0}^{\sqrt{2}} \\
= \left[ 0 + 4 \sin^{-1} (1) \right] - \left[ \frac{2}{2} \sqrt{(\sqrt{8})^2-(2)^2} + \left( \frac{\sqrt{8}}{2} \right)^2 \sin^{-1} \left( \frac{2}{\sqrt{8}} \right) \right] \\
= 4 \frac{\pi}{2} - 2 - 4 \sin^{-1} \left( \frac{1}{\sqrt{2}} \right) \\
= 2\pi - 2 - 4 \left( \frac{\pi}{4} \right) \\
= \pi - 2 \\
\therefore \text{Required area } = 2 + \pi - 2 \\
= \pi \\
|
| 4     | d)      | Solve \( y^3 \cdot \sec^2 x \, dx + (3y^2 \cdot \tan x - \sec^2 y) \, dy = 0 \) |

\[
y^3 \cdot \sec^2 x \, dx + (3y^2 \cdot \tan x - \sec^2 y) \, dy = 0 \\
\text{Comparing with } \int M \, dx + \int N \, dy = c \\
M = y^3 \cdot \sec^2 x \quad N = 3y^2 \cdot \tan x - \sec^2 y \\
\frac{\partial M}{\partial y} = 3y^2 \cdot \sec^2 x \quad \frac{\partial N}{\partial x} = 3y^2 \cdot \sec^2 x \\
\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \\
\text{D.E. is exact} \\
\therefore \text{Solution is} \\
\int y^3 \cdot \sec^2 x \, dx - \int \sec^2 y \, dy = c \\
y^3 \tan x - \tan y = c |

---

**Subject Code:** 17301
**Q. No. 4 e) Ans**

Solve \((x + y + 1)^2 \frac{dy}{dx} = 1\)

Put \(x + y + 1 = v\)

\[\therefore \frac{dy}{dx} = \frac{dv}{dx}\]

\[\therefore \frac{dy}{dx} = \frac{dv}{dx} - 1\]

\[\therefore v^2 \left(\frac{dv}{dx} - 1\right) = 1\]

\[\therefore v^2 \frac{dv}{dx} - v^2 = 1\]

\[\therefore v^2 \frac{dv}{dx} = 1 + v^2\]

\[\therefore \left(\frac{v^2}{1 + v^2}\right) dv = dx\]

\[\therefore \text{solution is}\]

\[\int \left(\frac{v^2}{1 + v^2}\right) dv = \int dx\]

\[\int \left(\frac{1 + v^2 - 1}{1 + v^2}\right) dv = \int dx\]

\[\int \left(\frac{1 - \frac{1}{1 + v^2}}{1 + v^2}\right) dv = \int dx\]

\[\therefore v - \tan^{-1} v = x + c\]

\[\therefore x + y + 1 - \tan^{-1} (x + y + 1) = x + c\]

\[\therefore y + 1 - \tan^{-1} (x + y + 1) = c\]

**Q. No. 4 f)** Verify that \(y = e^{\sin^{-1} x}\) is a solution of differential equation

\((1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0\)
Q. No. | Sub Q. N. | Answer | Marking Scheme
--- | --- | --- | ---
4 | Ans | Consider \( y = e^{m \sin^{-1} x} \) 
\[
\frac{dy}{dx} = e^{m \sin^{-1} x} m \frac{1}{\sqrt{1-x^2}} \\
\frac{dy}{dx} = \frac{my}{\sqrt{1-x^2}} \\
\sqrt{1-x^2} \frac{dy}{dx} = my \\
\text{Squaring,} \\
\therefore (1-x^2) \left( \frac{dy}{dx} \right)^2 = m^2 y^2 \\
\therefore (1-x^2) 2 \frac{dy}{dx} \frac{d^2 y}{dx^2} + \left( \frac{dy}{dx} \right)^2 (-2x) = m^2 \left( 2y \frac{dy}{dx} \right) \\
\therefore 2 \frac{dy}{dx} \left( 1-x^2 \right) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} = 2m^2 y \frac{dy}{dx} \\
\therefore (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = m^2 y \\
\therefore (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0
\]

**OR**

\[
(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0 \\
\text{Consider } y = e^{m \sin^{-1} x} \\
\therefore \frac{dy}{dx} = e^{m \sin^{-1} x} m \frac{1}{\sqrt{1-x^2}} \\
\therefore \frac{dy}{dx} = \frac{my}{\sqrt{1-x^2}} \\
\therefore \sqrt{1-x^2} \frac{dy}{dx} = my \\
\therefore \sqrt{1-x^2} \frac{d^2 y}{dx^2} + \frac{dy}{dx} \frac{1}{2\sqrt{1-x^2}} (-2x) = m \frac{dy}{dx} \\
\therefore \left( \sqrt{1-x^2} \right) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = m \sqrt{1-x^2} \frac{dy}{dx} \\
\therefore (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = m (my)
\]
<table>
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<td>[ (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = m^2 y ]</td>
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<td>[ (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0 ]</td>
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### 5

**Attempt any FOUR of the following:**

**a)** A Card is drawn at random from a well shuffled pack of 52 playing cards.

A= event that the card drawn is not a spade.
B= event that the card drawn is king.
Verify that events A and B are independent.

**Ans**

\[ n(s) = 52, \quad C_1 = 52 \]
\[ n(A) = 39, \quad C_1 = 39 \]
\[ n(B) = 4, \quad C_1 = 4 \]

\[ p(A) = \frac{n(A)}{n(s)} = \frac{39}{52} = \frac{3}{4} \]
\[ p(B) = \frac{n(B)}{n(s)} = \frac{4}{52} = \frac{1}{13} \]

Consider \[ p(A) \times p(B) = \frac{3}{4} \times \frac{1}{13} = \frac{3}{52} \]

\[ A \cap B = \text{Event that the card drawn is a king of heart or of diamond or of club} \]
\[ n(A \cap B) = 3, \quad C_1 = 3 \]

\[ p(A \cap B) = \frac{n(A \cap B)}{n(s)} = \frac{3}{52} \]

\[ \therefore p(A \cap B) = p(A) \times p(B) \]
\[ \therefore A \text{ and B Independent events.} \]

**b)** Assuming that the probability of a fatal accident during the year is \( \frac{1}{1200} \).

Calculate the probability that in a factory employing 300 workers there will be at least two fatal accidents in a year, \( e^{-0.25} = 0.7788 \).

**Ans**

\[ n = 300, \quad p = \frac{1}{1200} \]

\[ m = np = \frac{300}{1200} = 0.25 \]

\[ p(\text{at least two fatal accidents}) = 1 - [p(0) + p(1)] \]
### Q. No. 5

#### b)

\[ p(\text{at least two fatal accidents}) = 1 - \left( e^{-0.25 \cdot 0.25} + e^{-0.25} \cdot 0.025 \right) \]
\[ = 1 - \left( 0.7788 + 0.7788 \cdot 0.25 \right) \]
\[ = 0.0265 \]

#### c)

In certain examination 500 students appeared. Mean score is 68 with S.D 8. Find the number of students scoring

i) less than 50

\[ z = \frac{x - \bar{x}}{\sigma} = \frac{50 - 68}{8} = -2.25 \]

ii) more than 60

\[ z = \frac{x - \bar{x}}{\sigma} = \frac{60 - 68}{8} = -1 \]

**NOTE**: As the areas for the above problem are not given, the students cannot solve the problem completely. If students attempted to solve the problem and calculated up to value \( z \), full marks to be rewarded.

#### d)

Evaluate \( \int \frac{1}{4 + 5 \sin(2x)} \, dx \)

\[ \int \frac{1}{4 + 5 \sin(2x)} \, dx \]

Put \( \tan x = t \), \( dx = \frac{dt}{1 + t^2} \), \( \sin 2x = \frac{2t}{1 + t^2} \)

\[ = \int \frac{dt}{4 + 5 \left( \frac{2t}{1 + t^2} \right)} \]
### Question 5 (d)

\[
\int \frac{dt}{4 + 4t^2 + 10t} = \int \frac{dt}{4t^2 + 10t + 4}
\]

\[
= \int \frac{dt}{4t^2 + 10t + \frac{100}{16} - \frac{100}{16} + 4}
\]

\[
= \int \frac{dt}{\left(2t + \frac{5}{2}\right)^2 - \left(\frac{3}{2}\right)^2}
\]

\[
= \frac{1}{2 \times \frac{3}{2}} \log \left| \frac{2t + \frac{5}{2} - \frac{3}{2}}{2t + \frac{5}{2} + \frac{3}{2}} \right| \times \frac{1}{2} + c
\]

\[
= \frac{1}{6} \log \left| \frac{2t + 1}{2t + 4} \right| + c
\]

\[
= \frac{1}{6} \log \left| \frac{2 \tan x + 1}{2 \tan x + 4} \right| + c
\]

**OR**

\[
\int \frac{1}{4 + 5 \sin (2x)} \ dx
\]

Put \( \tan x = t \), \( dx = \frac{dt}{1 + t^2} \), \( \sin 2x = \frac{2t}{1 + t^2} \)

\[
= \int \frac{dt}{4 + 5 \left( \frac{2t}{1 + t^2} \right)}
\]

\[
= \int \frac{dt}{4 + 4t^2 + 10t}
\]

\[
= \int \frac{dt}{4t^2 + 10t + 4}
\]

\[
= \frac{1}{4} \int \frac{dt}{t^2 + \frac{5}{2}t + 1}
\]

\[
= \frac{1}{4} \int \frac{dt}{t^2 + \frac{5}{2}t + \frac{25}{16} - \frac{25}{16} + 1 - \frac{25}{16}}
\]
### SUMMER – 17 EXAMINATION

#### Model Answer

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</table>
| 5      | d)        | \[
\int \frac{dt}{4\left(\frac{t+5}{4}\right)^2 - \left(\frac{3}{4}\right)^2}
\]
= \[
\frac{1}{4} \times \frac{1}{3} \log \left| \frac{t+5}{4} - \frac{3}{4} \right|
\]  
= \[
\frac{1}{6} \log \left(\frac{2t+1}{2t+4}\right)
\]  
= \[
\frac{1}{6} \log \left(\frac{2 \tan x + 1}{2 \tan x + 4}\right)
\]  |
|        |           | ½     |

| e)     | Evaluate \(\int_{0}^{\frac{\pi}{2}} \frac{\sin(2x)}{4 - \sin^2 x} \) dx |
| Ans    | \[
\frac{\pi}{2} \sin(2x)_{0}^{\frac{\pi}{2}} dx
\]  
= \[
\frac{2}{4} \sin x \cos x \]  
= \[
\frac{1}{2} 2tdt_{0}^{\frac{1}{4} - t^2}
\]  
consider, \[
\frac{2t}{4 - t^2} = \frac{A}{2 + t} + \frac{B}{2 - t}
\]  
\[
\frac{2t}{(2 + t)(2 - t)} = \frac{A}{2 + t} + \frac{B}{2 - t}
\]  
\[
\therefore 2t = A(2 - t) + B(2 + t)
\]  
put \(t = -2\)  
\[
\therefore A = -1
\]  
put \(t = 2\)  
\[
B = 1
\]  |
|        | 04       |

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| 5 e) |  | \[
\int_0^2 \frac{2tdt}{4-t^2} = \int_0^1 \left( \frac{-1}{2+t} + \frac{1}{2-t} \right) dt \\
= \left[ -\log(2+t) - \log(2-t) \right]_0^1 \\
= (-\log 3 - \log 1) - (-\log 2 - \log 2) \\
= -\log 3 + \log 4 \\
\]

OR

\[
\int_0^\pi \frac{\sin(2x)}{4 - \sin^2 x} dx \\
= \frac{1}{2} \int_0^\pi \frac{2\sin x \cos x}{4 - \sin^2 x} dx \\
\]

Put \( \sin x = t \) when \( x = 0 \) \( t = 0 \)

\[
\cos x dx = dt \\
\]

when \( x = \frac{\pi}{2} \) \( t = 1 \)

\[
= \int_0^\frac{\pi}{2} \frac{2dt}{4-t^2} \\
= -\int_0^1 \frac{-2dt}{4-t^2} \\
= -\left[ \log(4-t^2) \right]_0^1 \\
= -\left[ \log(4-1^2) - \log(4-0^2) \right] \\
= -\log(3) + \log(4) \\
\]

f) Ans

Solve \((2x + e^x \log y) dx + \left( \frac{e^x}{y} + 1 \right) dy = 0\)

comparing with \( Mdx + Ndy = 0 \)

\[
\therefore M = 2x + e^x \log y \quad N = \frac{e^x}{y} + 1 \\
\]

\[
\frac{\partial M}{\partial y} = \frac{e^x}{y}, \quad \frac{\partial N}{\partial x} = \frac{e^x}{y} \]

\[
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \]

\[
\therefore \text{given D.E.equation is exact} \\
\]

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<td>( \frac{1}{2} + \frac{1}{2} )</td>
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### Q. No. 5

**: solution is**

\[
\int_{y-\text{constant}} Mdx + \int_{\text{terms free from } x} Ndy = c
\]

\[
\int_{y-\text{constant}} (2x + e^y \log y) dx + \int 1dy = c
\]

\[
\therefore \frac{2x^2}{2} + e^y \log y + y = c
\]

\[
\therefore x^2 + e^y \log y + y = c
\]

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### Q. No. 6

**Attempt any FOUR of the following:**

a) A Husband and wife appeared in an interview for two vacancies in an office

The Probability of husband's selection is \( \frac{1}{7} \) and that of wife's selection is \( \frac{1}{5} \)

Find the probability that

i) both of them are selected.

ii) only one of them is selected.

**Ans**

Given \( p(H) = \frac{1}{7} \) \( p(W) = \frac{1}{5} \)

\( p(H') = 1 - p(H) = \frac{6}{7} \) and \( p(W') = 1 - p(W) = \frac{4}{5} \)

i) \( p(H \cap W) = p(H) p(W) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35} \)

ii) \( p(H \cap W') + p(H' \cap W) = p(H) \times p(W') + p(H') \times p(W) \)

\[
= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} = \frac{2}{7}
\]

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b) A company manufactures electric motors. The probability that an electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors? \( (e^{-3} = 0.0498) \)

**Ans**

Given \( n = 300 \), \( p = 0.01 \)

\( m = np = 300 \times 0.01 = 3 \)

\[
P(5) = \frac{e^{-3}5^5}{5!}
\]

\[
= 0.1008
\]

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| 6      | c)        | In a test of 2000 electric bulbs it was found that the life of particular make was normally distributed with average life of 2040 hrs. and S.D.of 60 hrs. Estimate the number of bulbs likely to burn for (i) between 1920hrs. and 2160 hrs. (ii) more than 2150 hrs. Given that \( A(2) = 0.4772 \) \[ A(1.83) = 0.4664 \]
\[ \bar{x} = 2040, \sigma = 60 \]
\[ i) \quad x = 1920, \quad x = 2160 \]
\[ z = \frac{x - \bar{x}}{\sigma} = \frac{1920 - 2040}{60} = -2 \]
\[ z = \frac{x - \bar{x}}{\sigma} = \frac{2160 - 2040}{60} = 2 \]
\[ P(\text{Between 1920 hrs. and 2160 hrs.}) = A(-2) + A(2) = 0.4772 + 0.4772 \]
\[ = 0.9544 \]
Number of bulbs having life between 1920 hrs. and 2160 hrs.
\[ = 0.9544 \times 2000 \]
\[ = 1908.8 \approx 1909 \]
\[ ii) \quad x = 2150 \]
\[ z = \frac{x - \bar{x}}{S.D} = \frac{2150 - 2040}{60} = 1.83 \]
\[ P(\text{More than 2150 hrs.}) = 0.5 - A(1.83) \]
\[ = 0.5 - 0.4664 \]
\[ = 0.0336 \]
Number of bulbs having life more than 2150 hrs.
\[ = 0.0336 \times 2000 = 67.2 \approx 67 \]
|       | d)        | Find the maximum and minimum values of  \( 2x^3 - 3x^2 - 36x + 10 \)
\[ y = 2x^3 - 3x^2 - 36x + 10 \]
\[ \frac{dy}{dx} = 6x^2 - 6x - 36 \]
\[ \frac{d^2y}{dx^2} = 12x - 6 \]
\[ \frac{dy}{dx} = 0 \]
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</table>
| 6     | d)        | $6x^2 - 6x - 36 = 0$ or $x^2 - x - 6 = 0$  
  
  for $x = 3$  
  
  $\frac{d^2y}{dx^2} = 12(3) - 6 = 30$  
  
  $\therefore y$ is minimum at $x = 3$  
  
  $y_{\text{min}} = 2(3)^3 - 3(3)^2 - 36(3) + 10 = -71$  
  
  for $x = -2$  
  
  $\frac{d^2y}{dx^2} = 12(-2) - 6 = -30$  
  
  $\therefore y$ is maximum at $x = -2$  
  
  $y_{\text{max}} = 2(-2)^3 - 3(-2)^2 - 36(-2) + 10 = 54$ | 1 |
|       | e)        | Find the equation of the tangent and the normal to the curve  
  
  $2x^2 - xy + 3y^2 = 18$ at $(3,1)$  
  
  $2x^2 - xy + 3y^2 = 18$ at $(3,1)$  
  
  $\therefore 4x - x \left( \frac{dy}{dx} + y \right) + 6y \frac{dy}{dx} = 0$  
  
  $4x - x \frac{dy}{dx} - y + 6y \frac{dy}{dx} = 0$  
  
  $\frac{dy}{dx} = \frac{y - 4x}{6y - x}$  
  
  at $(3,1)$  
  
  $\frac{dy}{dx} = \frac{1 - 4(3)}{6(1) - 3} = \frac{-11}{3}$  
  
  $\therefore$ slope of tangent $= -\frac{11}{3}$  
  
  Equation of tangent is  
  
  $y - 1 = -\frac{11}{3}(x - 3)$  
  
  $3y - 3 = -11x + 33$  
  
  $11x + 3y - 36 = 0$  
  
  $\therefore$ slope of normal $= \frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$ | 2 |

Equation of normal is
## Model Answer

### Summer – 17 Examination

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| 6     | e)        | $y - 1 = \frac{3}{11}(x - 3)$
11$y - 11 = 3x - 9$
3$x - 11y + 2 = 0$ | 1 |
|       | f)        | $\text{Find by integration the area of the ellipse } 4x^2 + 9y^2 = 36$
$4x^2 + 9y^2 = 36$
$x^2 + \frac{y^2}{9} = 1$
$\therefore y^2 = \frac{4}{9}(9 - x^2)$
$\therefore y = \frac{2}{3}\sqrt{3^2 - x^2}$

Area $= 4 \int_{-3}^{3} y \, dx$

$= 4 \int_{0}^{3} \frac{2}{3}\sqrt{3^2 - x^2} \, dx$

$= \frac{8}{3} \int_{0}^{3} \sqrt{3^2 - x^2} \, dx$

$= \frac{8}{3} \left[ \frac{x}{2} \sqrt{(3)^2 - (x)^2} + \frac{3}{2} \sin^{-1} \left( \frac{x}{3} \right) \right]_{0}^{3}$

$= \frac{8}{3} \left[ \frac{3}{2} \sqrt{3^2 - (3)^2} + \frac{3}{2} \sin^{-1} \left( \frac{3}{3} \right) \right] - [0]$

$= \frac{8}{3} \left[ 0 + \frac{9}{2} \sin^{-1} (1) \right]$

$= \frac{8}{3} \frac{9}{2} \pi$

$= 6\pi$ | 04 |

### Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.

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