## 16117

## 3 Hours / 100 Marks

Seat No.

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Instructions: (1) All Questions are compulsory.
(2) Figures to the right indicate full marks.
(3) Assume suitable data, if necessary.
(4) Use of Non-programmable Electronic Pocket Calculator is permissible.
(5) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.
(6) Use of steam tables, logarithmic, Mollier's chart is permitted.

1. Attempt any TEN of the following :
(a) Find ' a ', if the tangent to the curve $\mathrm{y}=x^{2}+\mathrm{ax}$ at the origin is parallel to the line passing through $\mathrm{A}(-4,-3)$ and $\mathrm{B}(-2,5)$.
(b) Find Radius of curvature of $\mathrm{y}=x^{3}$ at $(1,1)$.
(c) Evaluate : $\int\left(\mathrm{e}^{x}+x^{\mathrm{e}}+\mathrm{e}^{\mathrm{e}}\right) \mathrm{d} x$.
(d) Evaluate : $\int \frac{1}{x+\sqrt{x}} \mathrm{~d} x$.
(e) Evaluate : $\int \sin ^{2} x \cdot \mathrm{~d} x$.
(f) Evaluate : $\int \sec ^{2} x \cdot x \cdot \mathrm{~d} x$.
P.T.O.
(g) Find ' $k$ ', if $\int_{0}^{1}\left(3 x^{2}+2 x+k\right) \mathrm{d} x=0$.
(h) Evaluate $\int \operatorname{cosec}^{2}\left(x^{\circ}\right) \mathrm{d} x$.
(i) Find the area under the parabola $\mathrm{y}^{2}=4 x$ bounded by the lines $x=0, \mathrm{y}=0$, $x=4$.
(j) Find order and degree of the differential equation $\frac{d^{2} y}{d x^{2}}=\left(y+\frac{d y}{d x}\right)^{3 / 2}$.
(k) Form a D.E. if $\mathrm{y}=\mathrm{A} \sin x+\mathrm{B} \cos x$.
(l) Form a differential equation, if $y=a x^{2}+b$.
(m) Find the probability of getting sum of numbers is 9 with two dice.
(n) An unbiased coin is tossed 3 times. Find the probability of getting two head.
2. Attempt any FOUR of the following :
(a) Find equation of tangent to the circle $x^{2}+y^{2}+6 x-6 y-7=0$ at a point it cuts the $x$-axis.
(b) Discuss maxima and minima of the function " $\tan x-2 x$ ".
(c) Find radius of curvature of the curve $\sqrt{x}+\sqrt{y}=1$ at $(1 / 4,1 / 4)$.
(d) Evaluate : $\int \tan ^{6} x \cdot \mathrm{~d} x$.
(e) Evaluate : $\int \cos (\log x) \cdot \mathrm{d} x$.
(f) Evaluate : $\int \frac{\sec ^{2} x}{(1+\tan x)(2+\tan x)} \mathrm{d} x$.
3. Attempt any FOUR of the following :
(a) Evaluate : $\int_{0}^{4} \frac{1}{\sqrt{4 x-x^{2}}} \mathrm{~d} x$.
(b) Evaluate : $\int_{0}^{\pi} \frac{x \cdot \sin x}{1+\cos ^{2} x} \mathrm{~d} x$.
(c) Find the area of the loop of the curve $y^{2}=x^{2}(1-x)$.
(d) Solve : $\frac{d y}{d x}=e^{(x-y)} \cdot x^{2}$.
(e) Solve : $(x-y) \frac{\mathrm{dy}}{\mathrm{d} x}=x+\mathrm{y}$.
(f) Solve : $(1+x) \frac{\mathrm{dy}}{\mathrm{d} x}-\mathrm{y}=\mathrm{e}^{3 x}(1+x)^{2}$.
4. Attempt any FOUR of the following :
(a) Evaluate : $\int_{1}^{4} \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}} \mathrm{~d} x$.
(b) Evaluate : $\int_{0}^{\pi / 2} \frac{\sin x \cdot \cos x}{\cos ^{2} x+3 \cos x+2} \mathrm{~d} x$.
(c) Find the area bounded by two parabola $y^{2}=2 x$ and $x^{2}=2 y$.
(d) Solve : $\left[4-\frac{y^{2}}{x^{2}}\right] d x+\frac{2 y}{x} d y=0$.
(e) Solve : $\left(y \cdot e^{x y}-2 y^{3}\right) d x+\left(x \cdot e^{x y}-6 x y^{2}-2 y\right) d y=0$
(f) Verify that $\mathrm{y}^{2}=\mathrm{a} x^{2}$ is a solution of $x\left(\frac{\mathrm{dy}}{\mathrm{d} x}\right)^{2}-2 \mathrm{y} \frac{\mathrm{dy}}{\mathrm{d} x}+\mathrm{a} x=0$.
5. Attempt any FOUR of the following :
(a) The probability that a student passes H.S.C. exam is $2 / 3$ and the probability that he passes both H.S.C. and I.I.T. entrance exam is $14 / 45$. The probability that he passes at least one exam is $4 / 5$. What is the probability that he passes the I.I.T. entrance exam?
(b) In 200 sets of tosses of 5 fair coins in how many ways you can expect
(i) at least two heads.
(ii) at the most two heads.
(c) If $5 \%$ of the electric bulbs manufacturing by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs.
(i) None is defective.
(ii) Five bulbs are defective (Given $\mathrm{e}^{-5}=0.007$ ).
(d) Evaluate : $\int \frac{x+1}{(x-1)^{2}} \mathrm{~d} x$.
(e) Evaluate : $\int_{0}^{\pi / 2} \sin 5 x \cdot \cos 3 x \cdot \mathrm{~d} x$.
(f) Evaluate : $\int \mathrm{e}^{x} \cdot \sin 4 x \mathrm{~d} x$.
6. Attempt any FOUR of the following :
(a) Two six faced unbiased dice are thrown. Find the probability that the sum of the numbers shown is 7 or product is 12 .
(b) In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5 . Assuming the distribution to be normal. Find
(i) How many students score between 12 and 15 ?
(ii) How many students score above 18 ?
(Given : $\mathrm{A}(0.8)=0.2881, \mathrm{~A}(0.4)=0.1554, \mathrm{~A}((1.6)=0.4452$.)
(c) A metal wire of 40 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.
(d) Find equation of normal and tangent to the curve $\mathrm{y}=4 \cdot x \cdot \mathrm{e}^{x}$ at the origin.
(e) If $\mathrm{P}(\mathrm{A})=\frac{1}{2}, \mathrm{P}\left(\mathrm{B}^{\prime}\right)=\frac{2}{3}, \mathrm{P}(\mathrm{A} \cup \mathrm{B})=\frac{2}{3}$, find $\mathrm{P}\left(\mathrm{A}^{\prime} \cap \mathrm{B}^{\prime}\right) \& \mathrm{P}(\mathrm{A} / \mathrm{B})$.
(f) The probability that a pen manufactured by a company will be defective is $1 / 10$. If 12 such pens are manufactured, find the probability that :
(i) Exactly two will be defective.
(ii) At least two will be defective.
(iii) None will be defective.
