MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION
(Autonomous)
(ISO/IEC - 27001 - 2013 Certified)

WINTER– 18 EXAMINATION

Subject Name: Engineering Mathematics **Model Answer** **Subject Code: 17216**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any TEN of the following:**	**20**
	a)	State whether the function is even or odd if $f\left(x\right)=\frac{a^{x}+a^{-x}}{2}$	**02**
	Ans	$f\left(-x\right)=\frac{a^{-x}+a^{-(-x)}}{2}$	1
		$f\left(-x\right)=\frac{a^{-x}+a^{x}}{2}$	½
		$\therefore f\left(-x\right)=f\left(x\right)$	
		$\therefore$ Given function is even	½
		---	
	b)	If $f\left(x\right)=x^{2}\frac{1}{x^{2}}$ show that, $f\left(x\right)+f\left(-x\right)=2f\left(x\right)$	**02**
	Ans	$f\left(x\right)=x^{2}\frac{1}{x^{2}}=1$	½
		$f\left(-x\right)=\left(-x\right)^{2}\frac{1}{\left(-x\right)^{2}}=1$	½
		$f\left(x\right)+f\left(-x\right)=1+1=2=2.1=2f\left(x\right)$	1
		---	
	c)	Seperate into real and immaginary part for, $\sin\left(x+iy\right)$	**02**
	Ans	$\sin\left(x+iy\right)=\sin x\cos iy+\cos x\sin iy$	½
		$\sin\left(x+iy\right)=\sin x\cosh y+i\cos x\sinh y$	1
		Real part $=\sin x\cosh y$	½
		Immaginary part $=\cos x\sinh y$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	d)	If $(3+i)x+(1-i)y=1+7i$ Find x and y	**02**
	Ans	$3x+ix+y-iy=1+7i$	
		$(3x+y)+i(x-y)=1+7i$	½
		$3x+y=1$ and $x-y=7$	½
		$x=2, y=-5$	1
		--	
	e)	Evaluate $\lim_{x\to1} \frac{x^2-1}{x-1}$	**02**
	Ans	$\lim_{x\to1} \frac{x^2-1}{x-1}$	
		$=\lim_{x\to1} \frac{(x-1)(x+1)}{x-1}$	1
		$=\lim_{x\to1} x+1=1+1=2$	1
		--	
	f)	Evaluate $\lim_{x\to0}\left[\frac{5\tan x+6x}{9x-2\sin x}\right]$	**02**
	Ans	$\lim_{x\to0}\left[\frac{5\tan x+6x}{9x-2\sin x}\right]$	
		$=\lim_{x\to0}\left[\frac{\frac{5\tan x+6x}{x}}{\frac{9x-2\sin x}{x}}\right]$	½
		$=\lim_{x\to0}\left[\frac{\frac{5\tan x}{x}+6}{9-\frac{2\sin x}{x}}\right]$	½
		$=\left[\frac{5+6}{9-2}\right]=\frac{11}{7}$	1
		--	
	g)	Evalute $\lim_{x\to0}\frac{a^x-b^x}{x}$	**02**
	Ans	$\lim_{x\to0}\frac{a^x-b^x}{x}$	

WINTER– 18 EXAMINATION

Subject Name: Engineering Mathematics **Model Answer** **Subject Code:** 17216

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	g)	$=\lim_{x\to 0}\frac{a^x-1-b^x+1}{x}$	½
		$=\lim_{x\to 0}\frac{\left(a^x-1\right)-\left(b^x-1\right)}{x}$	
		$=\left[\lim_{x\to 0}\frac{a^x-1}{x}\right]-\left[\lim_{x\to 0}\frac{b^x-1}{x}\right]$	½
		$=\log a-\log b=\log\left(\frac{a}{b}\right)$	1
		--	
	h)	If $y=x.\log x$ find $\frac{dy}{dx}$	**02**
	Ans	$y=x.\log x$	
		$\frac{dy}{dx}=x.\frac{1}{x}+\log x.1$	1½
		$\frac{dy}{dx}=1+\log x$	½
		--	
	i)	If $y=\cos\left(\log x\right)$ find $\frac{dy}{dx}$	**02**
	Ans	$y=\cos\left(\log x\right)$	
		$\frac{dy}{dx}=-\sin\left(\log x\right).\frac{1}{x}$	2
		--	
	j)	Differentiate $\sin x$ w.r.t. $'\log x'$	**02**
	Ans	$u=\sin x \quad , \; v=\log x$	
		$\frac{du}{dx}=\cos x \; , \; \frac{dv}{dx}=\frac{1}{x}$	½+½
		$\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\cos x}{\frac{1}{x}}$	1

Q. No.	Sub Q. N.	Answers	Marking Scheme
1.	j)	$= x\cos x$	
		--	
	k)	From the following system of Equations, $3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.52$ Find one iteration only using Gauss-seidal method.	**02**
	Ans	$3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.52$ $x=\frac{4.5-2y}{3}$ $y=\frac{5-2x+z}{3}$	1
		$z=\frac{0.52+y}{2}$ $x_1=1.5,\ y_1=0.667,\ z_1=0.594$	1
		--	
	l)	Show that the root of the equation $x^3-9x+1=0$ lies bet^n 2 and 3.	**02**
	Ans	$f(x)=x^3-9x+1$ $f(2)=2^3-9(2)+1=-9<0$	1
		$f(3)=3^3-9(3)+1=1>0$ root lies in 2 and 3	1
		--	
2.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Express $\frac{1}{2}+i\frac{\sqrt{3}}{2}$ in polar form.	**04**
	Ans	Let $z=\frac{1}{2}+i\frac{\sqrt{3}}{2}$ $x=\frac{1}{2},\ y=\frac{\sqrt{3}}{2}$ $r=\sqrt{x^2+y^2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$ $=1$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.		$\theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$	1
		$=\tan^{-1}\left(\sqrt{3}\right)=60^0$ or $\frac{\pi}{3}$	1
		In polar form, $z=r\left(\cos\theta+i\sin\theta\right)$ $=1\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$	1
	b)	Simplify using De Moivre's theorem $\frac{\left(\cos\frac{4}{3}\theta+i\sin\frac{4}{3}\theta\right)^3\left(\cos\frac{1}{2}\theta-i\sin\frac{1}{2}\theta\right)^2}{\left(\cos4\theta-i\sin4\theta\right)\left(\cos2\theta+i\sin2\theta\right)^3}$	**04**
	Ans	$\frac{\left(\cos\frac{4}{3}\theta+i\sin\frac{4}{3}\theta\right)^3\left(\cos\frac{1}{2}\theta-i\sin\frac{1}{2}\theta\right)^2}{\left(\cos4\theta-i\sin4\theta\right)\left(\cos2\theta+i\sin2\theta\right)^3}$	
		$=\frac{\left(\cos\theta+i\sin\theta\right)^4\left(\cos\theta+i\sin\theta\right)^{-1}}{\left(\cos\theta+i\sin\theta\right)^{-4}\left(\cos\theta+i\sin\theta\right)^6}$	2
		$=\left(\cos\theta+i\sin\theta\right)^{4-1+4-6}$	1
		$=\left(\cos\theta+i\sin\theta\right)^1$	½
		$=\cos\theta+i\sin\theta$	½
	c)	By using De Moivre's theorem find "cube root of unity".	**04**
	Ans	$x=1^{\frac{1}{3}}$ $\therefore x^3=1$ Put $x^3=z\ \therefore x=z^{\frac{1}{3}}$	
		$\therefore z=1+0i$ $\text{Re}(z)=1, \text{Im}(z)=0$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.		$r=\lvert z\rvert=\sqrt{1+0}=1$	1
		$\theta=\tan^{-1}\left(\frac{0}{1}\right)=0$	½
		$z=r(\cos\theta+i\sin\theta)$	
		$z=1(\cos 0+i\sin 0)$	
		In general polar form , $z=r\left(\cos\left(2\pi k+\theta\right)+i\sin\left(2\pi k+\theta\right)\right)$	
		$z=1(\cos 2\pi k+i\sin 2\pi k)$	
		$z^{\frac{1}{3}}=(\cos 2\pi k+i\sin 2\pi k)^{\frac{1}{3}}$	
		$z=\cos\left(\frac{2\pi k}{3}\right)+i\sin\left(\frac{2\pi k}{3}\right)\quad;\quad k=0,1,2$	½
		when $k=0$	
		$z_1=\cos 0+i\sin 0=1$	½
		when $k=1$	
		$z_2=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$	½
		when $k=2$	
		$z_3=\cos\left(\frac{4\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}\right)=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$	½
		--	
	d)	Show that $\sin 2\theta=2\sin\theta\cos\theta$ using Euler's form	**04**
	Ans	$\sin 2\theta=\left(\frac{e^{2i\theta}-e^{-2i\theta}}{2i}\right)------------(1)$	1
		$2\sin\theta\cos\theta=2\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)$	1
		$=\frac{1}{2i}\left(e^{i\theta}e^{i\theta}+e^{i\theta}e^{-i\theta}-e^{-i\theta}e^{i\theta}-e^{-i\theta}e^{-i\theta}\right)$ OR	
		$=\frac{1}{2i}\left(e^{2i\theta}+e^{0}-e^{0}-e^{-2i\theta}\right)$ $=\frac{1}{2i}\left(e^{(i\theta)^2}-e^{(-i\theta)^2}\right)$	1
		$=\left(\frac{e^{2i\theta}-e^{-2i\theta}}{2i}\right)-----------(2)$	1
		by (1) and (2) $\sin 2\theta=2\sin\theta\cos\theta$	
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	e)	If $f(x)=50\sin(100\pi x+0.4)$, prove that $f\left(\frac{1}{50}+x\right)=f(x)$	**04**
	Ans	$f\left(\frac{1}{50}+x\right)=50\sin\left(100\pi\left(\frac{1}{50}+x\right)+0.4\right)$	1
		$=50\sin(2\pi+100\pi x+0.4)$	1
		$=50\sin(100\pi x+0.4)$	1
		$=f(x)$	1
		--	
	f)	If $f(x)=\frac{x+3}{4x-5}$ and $t=\frac{3+5x}{4x-1}$ show that $f(t)=x$	**04**
	Ans	$f(t)=\frac{t+3}{4t-5}$	½
		$=\frac{\frac{3+5x}{4x-1}+3}{4\left(\frac{3+5x}{4x-1}\right)-5}$	1
		$=\frac{\frac{3+5x+3(4x-1)}{4x-1}}{\frac{4(3+5x)-5(4x-1)}{4x-1}}$	½
		$=\frac{3+5x+12x-3}{12+20x-20x+5}$	1
		$=\frac{17x}{17}=x$	1
		--	
3.		**Solve any <u>Four</u> of the following:**	**16**
	a)	If $f(x)=x^2+5$, find x if $f(x+2)=f(x-2)$	**04**
	Ans	$f(x+2)=(x+2)^2+5$	½
		$=x^2+4x+4+5$	
		$=x^2+4x+9$	1
		$f(x-2)=(x-2)^2+5$	½
		$=x^2-4x+4+5$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.		$= x^2 - 4x + 9$	1
		$\because f(x+2) = f(x-2)$	
		$\therefore x^2 + 4x + 9 = x^2 - 4x + 9$	½
		$8x = 0 \;\; \therefore x = 0$	½
		--	
	b)	If $f(x) = 16^x + \log_2 x$ find $f\left(\frac{1}{4}\right)$	**04**
	Ans	$f(x) = 16^x + \log_2 x$	
		$\therefore f\left(\frac{1}{4}\right) = (16)^{\frac{1}{4}} + \log_2\left(\frac{1}{4}\right)$	2
			2
		$= 0$	
		--	
	c)	Evaluate: $\lim\limits_{x\to 0} \frac{\sin 2x^0}{x}$	**04**
	Ans	$\lim\limits_{x\to 0} \frac{\sin 2x^0}{x}$	
		$= \lim\limits_{x\to 0} \frac{\sin 2\left(\frac{\pi x}{180}\right)}{x}$	1
		$= \lim\limits_{x\to 0} \frac{\sin\left(\frac{\pi x}{90}\right)}{\frac{\pi x}{90}} \times \frac{\pi}{90}$	2
		$= 1 \times \frac{\pi}{90} = \frac{\pi}{90}$	1
		--	
	d)	Evaluate: $\lim\limits_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$	**04**
	Ans	$\lim\limits_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$	
		$= \lim\limits_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \times \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$=\lim_{x\to 0}\dfrac{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	
		$=\lim_{x\to 0}\dfrac{(1+x)-(1-x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	1
		$=\lim_{x\to 0}\dfrac{1+x-1+x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	1
		$=\lim_{x\to 0}\dfrac{2x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	
		$=\lim_{x\to 0}\dfrac{2}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	½
		$=\dfrac{2}{\left(\sqrt{1+0}+\sqrt{1-0}\right)}$	½
		$=\dfrac{2}{2}=1$	½
		---	
	e)	Evaluate: $\lim_{x\to 0}\dfrac{12^x-4^x-3^x+1}{x^2}$	**04**
	Ans	$\lim_{x\to 0}\dfrac{12^x-4^x-3^x+1}{x^2}$	
		$=\lim_{x\to 0}\dfrac{4^x3^x-4^x-3^x+1}{x^2}$	½
		$=\lim_{x\to 0}\dfrac{4^x\left(3^x-1\right)-\left(3^x-1\right)}{x^2}$	½
		$=\lim_{x\to 0}\dfrac{\left(4^x-1\right)\left(3^x-1\right)}{x^2}$	1
		$=\lim_{x\to 0}\left[\dfrac{4^x-1}{x}\right]\lim_{x\to 0}\left[\dfrac{3^x-1}{x}\right]$	
		$=(\log 4)(\log 3)$	2
		---	
	f)	Evaluate: $\lim_{x\to\infty}\left[\dfrac{x+1}{x+2}\right]^x$	**04**

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	Ans	$\lim_{x\to\infty}\left[\frac{x+1}{x+2}\right]^x$	
		$=\lim_{x\to\infty}\left[\frac{\frac{x+1}{x}}{\frac{x+2}{x}}\right]^x$	1
		$=\lim_{x\to\infty}\left[\frac{\left(1+\frac{1}{x}\right)}{\left(1+\frac{2}{x}\right)}\right]^x$	1
		$=\lim_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)^x}{\left(1+\frac{2}{x}\right)^{\frac{x}{2}\times 2}}$	1
		$=\frac{e}{e^2}$	½
		$=\frac{1}{e}$	½
		--	
4.		**Solve any <u>FOUR</u> of the following:**	**04**
	a)	Using 1st principle of derivatives find derivatives of $f\left(x\right)=\log x$	
	Ans	$\frac{dy}{dx}=\lim_{h\to0}\frac{f\left(x+h\right)-f\left(x\right)}{h}$	
		$\frac{dy}{dx}=\lim_{h\to0}\frac{\log\left(x+h\right)-\log x}{h}$	1
		$\frac{dy}{dx}=\lim_{h\to0}\frac{\log\left(\frac{x+h}{x}\right)}{h}$	1
		$\frac{dy}{dx}=\lim_{h\to0}\log\left(\frac{x+h}{x}\right)^{\frac{1}{h}}$	½
		$\frac{dy}{dx}=\lim_{h\to0}\log\left(1+\frac{h}{x}\right)^{\frac{x}{h}\times\frac{1}{x}}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	a)	$= \log e^{\frac{1}{x}} = \frac{1}{x}$	1
		---	
	b)	If u and v are differentiable functions of x then prove that $\frac{d}{dx}[U.V] = u\frac{dv}{dx} + v\frac{du}{dx}$	**04**
	Ans	Let $y = uv$ Let $\delta u, \delta v, \delta y$ are small increments in u, v, y respectively corresponding to increment δx in x.	
		$\therefore y + \delta y = (u + \delta u)(v + \delta v)$	½
		$y + \delta y = uv + u\delta v + v\delta u + \delta u\delta v$ $\delta y = uv + u\delta v + v\delta u + \delta u\delta v - y$	
		$\delta y = uv + u\delta v + v\delta u + \delta u\delta v - uv$	½
		$\delta y = u\delta v + v\delta u + \delta u\delta v$	½
		$\because \delta u, \delta v$ are very small. $\therefore \delta u\delta v$ is negligible.	½
		$\therefore \delta y = u\delta v + v\delta u$	
		$\therefore \frac{\delta y}{\delta x} = \frac{u\delta v + v\delta u}{\delta x}$	½
		$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = u\lim_{\delta x \to 0}\frac{\delta v}{\delta x} + v\lim_{\delta x \to 0}\frac{\delta u}{\delta x}$	½
		$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$	1
		---	
	c)	Find $\frac{dy}{dx}$ if $y = x^x + a^x$	**04**
	Ans	$Let\ y = u + v$ $u = x^x \quad v = a^x$ $u = x^x$	
		$\log u = \log x^x$	½
		$\log u = x\log x$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.		$\frac{1}{u}\frac{du}{dx}=x\frac{1}{x}+\log x.1$	
		$\frac{du}{dx}=u\left(1+\log x\right)$	1
		$v=a^x$	
		$\frac{dv}{dx}=a^x\log a$	1
		$\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$	
		$=x^x\left(1+\log x\right)+a^x\log a$	1
		--	
	d)	Find $\frac{dy}{dx}$ if $y=\log\left[x^2-2x+\sin x\right]$	**04**
	Ans	$y=\log\left[x^2-2x+\sin x\right]$	
		$\frac{dy}{dx}=\frac{1}{x^2-2x+\sin x}\frac{d}{dx}\left(x^2-2x+\sin x\right)$	
		$\frac{dy}{dx}=\frac{1}{x^2-2x+\sin x}\left(2x-2+\cos x\right)$	4
		--	
	e)	Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left[\frac{x}{\left(1+12x^2\right)}\right]$	**04**
	Ans	$y=\tan^{-1}\left[\frac{x}{\left(1+12x^2\right)}\right]$	
		$y=\tan^{-1}\left[\frac{4x-3x}{\left(1+4x\times 3x\right)}\right]$	1
		$y=\tan^{-1}4x-\tan^{-1}3x$	1
		$\frac{dy}{dx}=\frac{1}{1+\left(4x\right)^2}\times 4-\frac{1}{1+\left(3x\right)^2}\times 3$	2
		$=\frac{4}{1+16x^2}-\frac{3}{1+9x^2}$	
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	f)	Find $\frac{dy}{dx}$ if $x^2+y^2=4xy$	**04**
	Ans	$x^2+y^2=4xy$	
		$2x+2y\frac{dy}{dx}=4\left(x\frac{dy}{dx}+y.1\right)$	1
		$2x+2y\frac{dy}{dx}=4x\frac{dy}{dx}+4y$	1
		$2y\frac{dy}{dx}-4x\frac{dy}{dx}=4y-2x$	
		$\frac{dy}{dx}(2y-4x)=4y-2x$	1
		$\frac{dy}{dx}=\frac{4y-2x}{2y-4x}$	1
		$\frac{dy}{dx}=\frac{2y-x}{y-2x}$	

5.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Evaluate: $\lim_{x\to 0}\frac{\log(a+x)-\log a}{x}$	**04**
	Ans	$\lim_{x\to 0}\frac{\log(a+x)-\log a}{x}$	
		$=\lim_{x\to 0}\frac{\log\left(\frac{a+x}{a}\right)}{x}$	1
		$=\lim_{x\to 0}\log\left(\frac{a+x}{a}\right)^{\frac{1}{x}}$	1
		$=\lim_{x\to 0}\log\left(1+\frac{x}{a}\right)^{\frac{a}{x}\times\frac{1}{a}}$	1
		$=\log e^{\frac{1}{a}}=\frac{1}{a}$	1
		---	
	b)	Evaluate: $\lim_{x\to 0}\frac{3\sin x-\sin 3x}{x^3}$	**04**
	Ans	$\lim_{x\to 0}\frac{3\sin x-\sin 3x}{x^3}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.		$=\lim_{x\to 0}\frac{3\sin x-\left(3\sin x-4\sin^3 x\right)}{x^3}$	1
		$=\lim_{x\to 0}\frac{3\sin x-3\sin x+4\sin^3 x}{x^3}$	½
		$=\lim_{x\to 0}\frac{4\sin^3 x}{x^3}$	½
		$=\lim_{x\to 0}4\left(\frac{\sin x}{x}\right)^3=4\times 1^3=4$	2
		--	
	c)	Using Bisection method find approximate root of the equation $x^3-4x-9=0$	**04**
	Ans	*Let* $f\left(x\right)=x^3-4x-9$	
		$f\left(2\right)=-9<0$	
		$f\left(3\right)=6>0$	
		$\therefore$ the root is in $\left(2,3\right)$	1
		$x_1=\frac{a+b}{2}=\frac{2+3}{2}=2.5$	1
		$f\left(2.5\right)=-3.375<0$	
		$\therefore$ the root is in (2.5,3)	
		$x_2=\frac{x_1+b}{2}=\frac{2.5+3}{2}=2.75$	1
		$f\left(2.75\right)=0.797>0$	
		the root is in $\left(2.5,2.75\right)$	
		$x_3=\frac{x_1+x_2}{2}=\frac{2.5+2.75}{2}=2.625$	1
		OR	
		Let $f\left(x\right)=x^3-4x-9$	
		$f\left(2\right)=-9<0$	
		$f\left(3\right)=6>0$	
		$\therefore$ the root is in $\left(2,3\right)$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.			

Iteration	a	b	$x=\frac{a+b}{2}$	$f(x)$
I	2	3	2.5	-3.375
II	2.5	3	2.75	0.797
III	2.5	2.75	2.625	----

Marks: 1+1+1

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d) Find approximate root of the equation $x.\log_e x = 1.2$ by using bisection method. **04**

Ans

$x.\log_e x = 1.2$

$x.\log_e x - 1.2 = 0$

$f(x) = x.\log_e x - 1.2$

$f(1) = -1.2 < 0$

$f(2) = 0.186 > 0$

$\therefore$ the root is in $(1,2)$ — 1

$x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$ — 1

$f(1.5) = -0.592 < 0$

$\therefore$ the root is in (1.5,2)

$x_2 = \frac{x_1+b}{2} = \frac{1.5+2}{2} = 1.75$ — 1

$f(1.75) = -0.221 < 0$

the root is in $(1.75,2)$

$x_3 = \frac{x_2+b}{2} = \frac{1.75+2}{2} = 1.875$ — 1

OR

Let $f(x) = x\log_e x - 1.2$

$f(1) = -1.2 < 0$

$f(2) = 0.186 > 0$

$\therefore$ the root is in $(1,2)$ — 1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.			

Iteration	a	b	$x=\frac{a+b}{2}$	$f(x)$
I	1	2	1.5	-0.592
II	1.5	2	1.75	-0.221
III	1.75	2	1.875	----

1+1+1

---

e) Find the root of the equation $x^2+x-3=0$ using Regula-Falsi method. **04**

Ans

Let $f(x)=x^2+x-3$

$f(1)=-1<0$

$f(2)=3>0$

$\therefore$ the root is in $(1,2)$ 1

$$x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}$$

$$=\frac{1(3)-2(-1)}{3-(-1)}=1.25$$ 1

$f(x_1)=-0.188<0$

$\therefore$ the root is in $(1.25,2)$

$$x_2=\frac{1.25(3)-2(-0.188)}{3+0.188}=1.294$$ 1

$f(x_2)=-0.032<0$

$\therefore$ the root is in $(1.294,2)$

$$x_3=\frac{1.294(3)-2(-0.032)}{3+0.032}=1.301$$ 1

OR

Let $f(x)=x^2+x-3$

$f(1)=-1<0$

$f(2)=3>0$

$\therefore$ the root is in $(1,2)$

1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.			

a	B	$f(a)$	$f(b)$	$x=\dfrac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x)$
1	2	-1	3	1.25	-0.188
1.25	2	-0.188	3	1.294	-0.032
1.294	2	-0.032	3	1.301	---

1+1+1

---

f) Use Newton-Raphson method to find root of equation $x^2+x-3=0$ (up to three iterations) **04**

Ans Let $f(x)=x^2+x-3$

$f(1)=-1<0$

$f(2)=3>0$ ½

$f'(x)=2x+1$ ½

Initial root $x_0=1$

$\therefore f'(1)=3$

$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=1-\dfrac{f(1)}{f'(1)}=1.333$ 1

$x_2=1.333-\dfrac{f(1.333)}{f'(1.333)}=1.303$ 1

$x_3=1.303-\dfrac{f(1.303)}{f'(1.303)}=1.303$ 1

OR

Let $(x)=x^2+x-3$

$f(1)=-1<0$

$f(2)=3>0$ ½

$f'(x)=2x+1$ ½

Initial root $x_0=1$

Q. No.	Sub Q.N.	Answers	Marking Scheme
		$x_i = x - \frac{f(x)}{f'(x)} = x - \frac{x^2+x-3}{2x+1}$	
		$= \frac{2x^2+x-x^2-x+3}{2x+1}$	
		$= \frac{x^2+3}{2x+1}$	
		$x_1 = 1.333$	1
		$x_2 = 1.303$	1
		$x_3 = 1.303$	1
		--	
6.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	If $y = 2\cos[\log x] + 3\sin[\log x]$ prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$	**04**
	Ans	$y = 2\cos[\log x] + 3\sin[\log x]$	
		$\frac{dy}{dx} = -2\sin[\log x]\times\frac{1}{x} + 3\cos[\log x]\times\frac{1}{x}$	1
		$x\frac{dy}{dx} = -2\sin[\log x] + 3\cos[\log x]$	½
		$x\frac{d^2y}{dx^2} + \frac{dy}{dx}.1 = -2\cos[\log x]\times\frac{1}{x} - 3\sin[\log x]\times\frac{1}{x}$	1
		$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -\left(2\cos[\log x] + 3\sin[\log x]\right)$	½
		$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$	½
		$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$	½
		--	
	b)	If $x = a[\theta - \sin\theta]$ and $y = a[1-\cos\theta]$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$	**04**
	Ans	$x = a[\theta - \sin\theta], y = a[1-\cos\theta]$	
		$\therefore \frac{dy}{d\theta} = a\sin\theta$	½
		$\therefore \frac{dx}{d\theta} = a(1-\cos\theta)$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\therefore \frac{dy}{dx}=\frac{a\sin\theta}{a(1-\cos\theta)}=\frac{\sin\theta}{1-\cos\theta}$	½
		$\therefore \frac{dy}{dx}=\frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\sin^2\left(\frac{\theta}{2}\right)}=\cot\left(\frac{\theta}{2}\right)$	½
		$\therefore \frac{d^2y}{dx^2}=-\cos ec^2\left(\frac{\theta}{2}\right)\times\frac{1}{2}\times\frac{d\theta}{dx}$	½
		$\therefore \frac{d^2y}{dx^2}=-\cos ec^2\left(\frac{\theta}{2}\right)\times\frac{1}{a(1-\cos\theta)}\times\frac{1}{2}$	
		$\therefore \frac{d^2y}{dx^2}=-\frac{1}{4a}\cos ec^4\left(\frac{\theta}{2}\right)$	½
		$at \quad \theta=\frac{\pi}{4}$	
		$\frac{dy}{dx}=\cot\left(\frac{\pi}{8}\right)=2.414$	½
		$\therefore \frac{d^2y}{dx^2}=-\frac{1}{4a}\cos ec^4\left(\frac{\pi}{8}\right)=-\frac{1}{a}\times 11.657$	½
		--	
	c)	Solve the following equations by Gauss elimination method.	
		$4x+y+2z=12 \;,\; -x+11y+4z=33 \;,\; 2x-3y+8z=20$	**04**
	Ans	$4x+y+2z=12$	
		$-x+11y+4z=33$	
		$2x-3y+8z=20$	
		$4x+y+2z=12$ and $2x-3y+8z=20$	
		$-4x+44y+16z=132$ and $-2x+22y+8z=66$	
		+ ______ + ______	
		$45y+18z=144$ and $19y+16z=86$	
		$\therefore 5y+2z=16$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.		$40y+16z=128$	
		$19y+16z=86$	
		$-$ ___________	
		$21y=42$	
		$\therefore y=2$	1
		$z=3$	1
		$x=1$	1
		Note: *In the above solution, first x is eliminated and then z is eliminated to find the value of y first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking.*	
		--	
	d)	Solve the following equtions by using Jacobi's method	**04**
		$20x+y-2z=17$, $3x+20y-z+18=0$, $2x-3y+20z=25$	
	Ans	$20x+y-2z=17$, $3x+20y-z=-18$, $2x-3y+20z=25$	
		$x=\frac{1}{20}(17-y+2z)$	
		$y=\frac{1}{20}(-18-3x+z)$	1
		$z=\frac{1}{20}(25-2x+3y)$	
		Starting with $x_0=y_0=z_0=0$	
		$x_1=0.85$	
		$y_1=-0.9$	1
		$z_1=1.25$	
		$x_2=1.02$	
		$y_2=-0.965$	1
		$z_2=1.03$	
		$x_3=1.001$	
		$y_3=-1.002$	1
		$z_3=1.003$	
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	e)	Solve the following equations by Gauss elimination method. $2x+3y+z=13$, $x-y-2z+1=0$, $3x+y+4z=15$	**04**
	Ans	$2x+3y+z=13$ $x-y-2z=-1$ $3x+y+4z=15$	
		$x-y-2z=-1$ $3x+y+4z=15$ $+$ ___________ $4x+2z=14$ $2x+z=7$ and $3x-3y-6z=-3$ $2x+3y+z=13$ $+$ ___________ $5x-5z=10$ $x-z=2$	1
		$2x+z=7$ $x-z=2$ $+$ ___________ $3x=9$ $\therefore x=3$	1
		$z=1$	1
		$\mathrm{y}=2$	1
		***Note:** In the above solution, first x is eliminated and then z is eliminated to find the value of y first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking.* --	
	f)	Solve the following equations using Gauss-Seidal method. $10x+y+z=12$, $x+10y+z=12$, $x+y+10z=12$	**04**
	Ans	$x=\frac{1}{10}(12-y-z)$ $y=\frac{1}{10}(12-x-z)$ $z=\frac{1}{10}(12-x-y)$	1
		Starting with $x_0=y_0=z_0=0$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	f)	$x_1 = 1.2$ $y_1 = 1.08$ $z_1 = 0.972$	1
		$x_2 = 0.995$ $y_2 = 1.003$ $z_2 = 1$	1
		$x_3 = 1$ $y_3 = 1$ $z_3 = 1$	1

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.
