Instructions –
1. All Questions are Compulsory.
2. Answer each next main Question on a new page.
3. Figures to the right indicate full marks.
4. Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

1. Attempt any TEN of the following: 20

   a) If \( \frac{10}{3+4i} = a + ib \) find \( a \) and \( b \).

   b) If \( z = 3 + 4i \) Find \( z^2 - 6z + 25 \)

   c) If \( f(x) = x^2 + 6x + 10 \) Find \( f(2) + f(-2) \)

   d) If \( f(x) = \frac{a^x + a^{-x}}{2} \) prove that the function is even function.

   e) Evaluate \( \lim_{x \to 3} \frac{x^2 - 9}{x-3} \)

   f) Evaluate \( \lim_{x \to 0} \frac{1-\cos x}{x^2} \)

   g) Evaluate \( \lim_{x \to \infty} \left( \frac{x}{x+1} \right)^x \)

   h) If \( y = e^x \sin x \) find \( \frac{dy}{dx} \).

P.T.O.
i) If \( y = \tan^{-1}\left(\frac{a+x}{1-ax}\right) \) Find \( \frac{dy}{dx} \).

j) If \( x = a \sec t \) and \( y = b \tan t \) then find \( \frac{dy}{dx} \).

k) Prove that the root of equation \( x^3 - x - 4 = 0 \) lies between 0 and 2.

l) Find the first iteration by using Jacobi’s method for the following equation.
\[ 4x + y + 3z = 17, \quad x + 5y + z = 14 \quad \text{and} \quad 2x - y + 8z = 12. \]

2. **Attempt any **FOUR** of the following:**

a) If \( f(x) = \tan x \), prove that \( f(2x) = \frac{2f(x)}{1-f^2(x)} \)

b) Simplify using De-moiver’s theorem
\[
\frac{(\cos 3\theta + i\sin 3\theta)^4}{(\cos 5\theta - i\sin 5\theta)^{\frac{4}{5}}} \frac{(\cos 5\theta - i\sin 5\theta)^{\frac{4}{5}}}{(\cos \frac{4}{5}\theta + i\sin \frac{4}{5}\theta)^5} \frac{(\cos \frac{4}{5}\theta + i\sin \frac{4}{5}\theta)^5}{(\cos \frac{4}{5}\theta + i\sin \frac{4}{5}\theta)^{10}}
\]

c) Separate into real and imaginary part of \( \sin(x + iy) \)

d) Express in Polar form \( 1-\sqrt{3}i \)

e) Show that \( (1+i)^{12} + (1-i)^{12} = -128 \)

f) If \( f(x) = \log\left(\frac{x+1}{x-1}\right) \) prove that \( f\left(\frac{1+x^2}{2x}\right) = 2f(x) \)
3. Attempt any FOUR of the following: 16

a) Find \( f(t) \), if \( f(x) = \frac{2x + 5}{3x - 4} \) and \( t = \frac{5 + 4x}{3x - 2} \).

b) Evaluate \( \lim_{{x \to 0}} \frac{3^x + 3^{-x} - 2}{x^2} \).

c) If \( f(x) = x^2 - 3x + 4 \) solve \( f(1-x) = f(2x + 1) \).

d) Evaluate \( \lim_{{x \to 3}} \frac{x^3 - 7x^2 + 15x - 9}{x^3 - 4x^2 - 3x + 18} \).

e) Evaluate \( \lim_{{x \to \infty}} \sqrt{x^2 + x + 1} - x \).

f) Evaluate \( \lim_{{x \to 0}} \frac{2 \sin x - \sin 2x}{x^3} \).

4. Attempt any FOUR of the following: 16

a) Find \( \frac{dy}{dx} \) if \( y = \cos^{-1}(2x^2 - 1) \).

b) If \( x^2 + y^2 - xy = 0 \) find \( \frac{dy}{dx} \).

c) If \( x = a(1 + \cos \theta) \) \( y = a(1 - \cos \theta) \) find \( \frac{dy}{dx} \).

d) Using first principle find derivate of \( f(x) = \tan x \).

e) If \( u \) and \( v \) are differentiable functions of \( x \) and \( y = u + v \) then prove that \( \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \).

f) If \( x^y = e^{x-y} \) prove that \( \frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2} \).

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5. Attempt any **FOUR** of the following: 16

a) Evaluate \( \lim_{x \to 3} \frac{\log x - \log 3}{x-3} \)

b) Evaluate \( \lim_{x \to 0} \frac{(5^x-1)\tan x}{\sqrt{x^2+16}-4} \)

c) Find the root of the equation \( x^3-9x+1 = 0 \) which lies between 2 and 3 using Regula falsi method.

d) Find a root of \( x^3-9x^2-18 = 0 \) by Newton-Raphson method. (carry out 3 iterations)

e) Using Newton-Raphson method, find approximate value of \( \sqrt{10} \) (carry out 3 iterations)

f) Find the root of equation \( x^3-4x+1 = 0 \) using bisection method (carry out 3 iterations)

6. Attempt any **FOUR** of the following: 16

a) Find \( \frac{d^2y}{dx^2} \) if \( x = a\cos \theta \quad y = a\sin \theta \).

b) Solve the following equation by Gauss elimination method
\[
2x + y + z = 10, \quad 3x + 2y + 3z = 18 \quad \text{and} \quad x + 4y + 9z = 16.
\]

c) Solve the following equation by Gauss-Sedial method taking two iterations.
\[
10x + y + z = 12, \quad 2x + 10y + z = 13 \quad \text{and} \quad 2x + 2y + 10z = 14.
\]

d) Solve the following equation by Jacobi’s method by performing two iteration’s only
\[
15x + 2y + z = 18, \quad 2x + 20y - 3z = 19 \quad \text{and} \quad 3x - 6y + 25z = 22.
\]

e) Solve by Jacobi method, carry out two iterations
\[
10x + y + 2z = 13; \quad 3x + 10y + z = 14; \quad 2x + 3y + 10z = 15.
\]

f) If \( y = e^{m \sin^{-1}x} \) prove that \( (1-x^2)\frac{d^2y}{dx^2} - x \frac{dy}{dx} = m^2y = 0 \)