WINTER– 18 EXAMINATION

Subject Name: Basic Mathematics **Model Answer** **Subject Code: 17104**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any TEN of the following:**	**20**
	a)	Find the value of x if $\begin{bmatrix} x & 4 & -4 \\ 3 & -2 & 1 \\ -2 & -4 & 4 \end{bmatrix} = 0$	**02**
	Ans	$x\begin{vmatrix} -2 & 1 \\ -4 & 4 \end{vmatrix} - 4\begin{vmatrix} 3 & 1 \\ -2 & 4 \end{vmatrix} - 4\begin{vmatrix} 3 & -2 \\ -2 & -4 \end{vmatrix} = 0$	
		$\therefore x(-8+4) - 4(12+2) - 4(-12-4) = 0$	1
		$\therefore -4x - 56 + 64 = 0$	
		$\therefore -4x = -8$	
		$\therefore x = 2$	1
		Note: "If Students attempted to solve the problem by assuming appropriate value. Consider and Give appropriate marks."	
		---	
	b)	If $A = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$ Find the matrix B such that $2A + B = 0$	**02**
	Ans	$2A + B = 0 \qquad \therefore B = -2A$	
		$\therefore B = -2\begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$	1
		$\therefore B = \begin{bmatrix} -6 & 2 \\ 0 & -8 \end{bmatrix}$	1

WINTER– 18 EXAMINATION

Subject Name: Basic Mathematics **Model Answer** **Subject Code: 17104**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	If $A=\begin{bmatrix}3 & 4 & -2\\ 2 & 1 & 0\end{bmatrix}$, $B=\begin{bmatrix}2 & -1\\ 3 & 4\\ -0 & 2\end{bmatrix}$ find AB	**02**
	Ans	$AB=\begin{bmatrix}3 & 4 & -2\\ 2 & 1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & 4\\ 0 & 2\end{bmatrix}$	
		$\therefore AB=\begin{bmatrix}6+12+0 & -3+16-4\\ 4+3+0 & -2+4+0\end{bmatrix}$	1
		$\therefore AB=\begin{bmatrix}18 & 9\\ 7 & 2\end{bmatrix}$	1
		---	
	d)	Resolve into partial fraction $\frac{x}{x^2-x-2}$	**02**
	Ans	$\frac{x}{x^2-x-2}=\frac{x}{(x-2)(x+1)}$	½
		$\therefore \frac{x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$	
		$\therefore x=A(x+1)+B(x-2)$	½
		$\therefore$ put $x=2$ $\therefore$ $A=2/3$	½
		Put $x=-1 \therefore B=1/3$	
		$\therefore \frac{x}{(x-2)(x+1)}=\frac{2/3}{x-2}+\frac{1/3}{x+1}$	½
		---	
	e)	Define Compound angle	**02**
	Ans	An angle obtained by algebraic sum or difference of two or more angles is called compound angle.	2
		---	
	f)	Prove that $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$	**02**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	f) Ans	$\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}.\tan\theta}$	1
		$=\frac{1-\tan\theta}{1+1.\tan\theta}$	½
		$=\frac{1-\tan\theta}{1+\tan\theta}$	
		$=R.H.S.$	½
	g) Ans	Prove that $\cos 2A=\cos^2 A-\sin^2 A$	**02**
		$\cos 2A=\cos(A+A)$	½
		$=\cos A\cdot\cos A-\sin A\cdot\sin A$	1
		$=\cos^2 A-\sin^2 A$	½
	h) Ans	If $2\cos 70^0\cdot\sin 50^0=\sin A-\sin B$. find the angles A and B	**02**
		$2\cos 70^0\cdot\sin 50^0=\sin(70^0+50^0)-\sin(70^0-50^0)$	1
		$=\sin 120^0-\sin 20^0$	
		$=\sin A-\sin B$	
		$\therefore\ A=120^0$ and $B=20^0$	1
	i) Ans	Show that $2x+y+3=0$ *and* $x-2y-1=0$ are perpendicular lines.	**02**
		Slope of line $2x+y+3=0$ is $m_1=\frac{-a}{b}=\frac{-2}{1}=-2$	½
		Slope of line $x-2y-1=0$ is $m_2=\frac{-a}{b}=\frac{-1}{-2}=\frac{1}{2}$	½
		$\therefore m_1\cdot m_2=-2\times\frac{1}{2}$	½
		$\therefore m_1\cdot m_2=-1$	½
		Lines are perpendicular.	

WINTER– 18 EXAMINATION

Subject Name: Basic Mathematics **Model Answer** **Subject Code: 17104**

Q. No.	Sub Q. N.	Answers	Marking Scheme
1.	j)	Find the equation of the line Passing through (4,-5) and having slope $\frac{-2}{3}$	**02**
	Ans	The equation of line is $y-y_1=m(x-x_1)$	
		$\therefore y+5=\frac{-2}{3}(x-4)$	1
		$\therefore 3y+15=-2x+8$	
		$\therefore 2x+3y+7=0$	1
	k)	Find range and co-efficient of range of the following:	**02**

x_i	10	20	30	40	50
f_i	7	5	3	2	1

Ans	Range $=$ Largest value of x_i $-$ Smallest value of x_i	
	$=L-S$	
	$=50-10$	
	$=40$	1
	Co-efficient of range $=\frac{L-S}{L+S}$	
	Co-efficient of range $=\frac{50-10}{50+10}$	
	$=\frac{2}{3}$ or 0.667	1
l)	If the mean is 82.5 , S.D. is 7.2. Find coefficient of variance.	**02**
Ans	$C.V.=\frac{S.D}{Mean}\times 100$	
	$\therefore C.V=\frac{7.2}{82.5}\times 100$	1
	$\therefore C.V=8.727$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.		**Solve any FOUR of the following :**	**16**
	a)	Solve by using Cramer's rule $x+y=0,\ \ y+z=2,\ \ x+z=4$	**04**
	Ans	$D=\begin{vmatrix}1&1&0\\0&1&1\\1&0&1\end{vmatrix}=1(1-0)-1(0-1)+0=1+1=2$	1
		$D_x=\begin{vmatrix}0&1&0\\2&1&1\\4&0&1\end{vmatrix}=0-1(2-4)+0=2$ $\therefore x=\frac{D_x}{D}=\frac{2}{2}=1$	1
		$D_y=\begin{vmatrix}1&0&0\\0&2&1\\1&4&1\end{vmatrix}=1(2-4)-0+0=-2$ $\therefore y=\frac{D_y}{D}=\frac{-2}{2}=-1$	1
		$D_z=\begin{vmatrix}1&1&0\\0&1&2\\1&0&4\end{vmatrix}=1(4-0)-1(0-2)+0=4+2=6$ $\therefore z=\frac{D_z}{D}=\frac{6}{2}=3$	1
	b)	If $A=\begin{bmatrix}2&-1&1\\-2&3&-2\\-4&4&-3\end{bmatrix}$ Find A^2	**04**
	Ans	$A^2=\begin{bmatrix}2&-1&1\\-2&3&-2\\-4&4&-3\end{bmatrix}\begin{bmatrix}2&-1&1\\-2&3&-2\\-4&4&-3\end{bmatrix}$ $\therefore A^2=\begin{bmatrix}4+2-4&-2-3+4&2+2-3\\-4-6+8&2+9-8&-2-6+6\\-8-8+12&4+12-12&-4-8+9\end{bmatrix}$	2
		$\therefore A^2=\begin{bmatrix}2&-1&1\\-2&3&-2\\-4&4&-3\end{bmatrix}$	2

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	c)	If A = $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$,B= $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ prove that $A(B+C) = AB + BC$	**04**
	Ans	$B + C = \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$	1
		$\therefore A(B+C) = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} -1+8 & 2+6 \\ 2+12 & -4+9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$	1
		$AB = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2+4 & 1+6 \\ -4+6 & -2+9 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix}$	½
		$AC = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -3+4 & 1+0 \\ 6+6 & -2+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$	½
		$AB + AC = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$	1
		$\therefore A(B+C) = AB + AC$	
		<u>Note</u>: "If Students attempted to solve the problem by assuming appropriate value. Consider and Give appropriate marks."	
		---	
	d)	Express the matrix A as sum of symmetric and skew symmetric matrix of $A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$	**04**
	Ans	$A + A^T = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 5 \\ 7 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 9 & 6 & 4 \\ 6 & 4 & 10 \end{bmatrix}$	$1\frac{1}{2}$
		$A - A^T = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 5 \\ 7 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 5 & -4 \\ -5 & 0 & 4 \\ 4 & -4 & 0 \end{bmatrix}$	$1\frac{1}{2}$
		$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$	

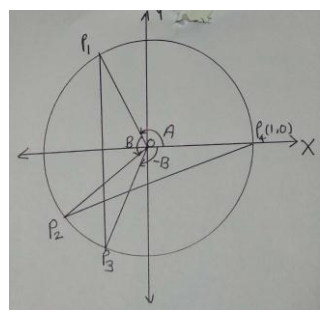
Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	$\therefore A=\frac{1}{2}\begin{bmatrix}-2 & 9 & 6\\ 9 & 6 & 4\\ 6 & 4 & 10\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0 & 5 & -4\\ -5 & 0 & 4\\ 4 & -4 & 0\end{bmatrix}=\underbrace{\begin{bmatrix}-1 & \frac{9}{2} & 3\\ \frac{9}{2} & 3 & 2\\ 3 & 2 & 5\end{bmatrix}}_{Symmetric}+\underbrace{\begin{bmatrix}0 & \frac{5}{2} & -2\\ -\frac{5}{2} & 0 & 2\\ 2 & -2 & 0\end{bmatrix}}_{Skew\ Symmetric}$	1
		---	
	e)	Resolve in to partial fraction $\frac{x^2}{\left(x^4+x^2-2\right)}$	**04**
	Ans	$\frac{x^2}{\left(x^4+x^2-2\right)}$	
		Put $x^2=y$	½
		$\therefore \frac{x^2}{\left(x^4+x^2-2\right)}=\frac{y}{y^2+y-2}$	
		$\therefore \frac{y}{y^2+y-2}=\frac{A}{y+2}+\frac{B}{y-1}$	½
		$\therefore y=A(y-1)+B(y+2)$	
		Put $y=-2$ $\therefore A=2/3$	1
		Put $y=1$ $\therefore B=1/3$	1
		$\therefore \frac{y}{y^2+y-2}=\frac{2/3}{y+2}+\frac{1/3}{y-1}$	½
		Re *place y by* x^2	
		$\therefore \frac{x^2}{\left(x^4+x^2-2\right)}=\frac{2/3}{x^2+2}+\frac{1/3}{x^2-1}$	½
		---	
	f)	Resolve into partial fraction $\frac{2x+1}{x^2(x+1)}$	**04**
	Ans	Let $\frac{2x+1}{x^2(x+1)}=\frac{A}{x+1}+\frac{B}{x}+\frac{C}{x^2}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	f)	$\therefore 2x+1 = A(x^2)+B(x)(x+1)+C(x+1)$	
		Put $x=-1 \quad \therefore A=-1$	1
		Put $x=0 \quad \therefore C=1$	1
		Put $x=1 \quad \therefore B=1$	1
		$\therefore \dfrac{2x+1}{x^2(x+1)} = \dfrac{-1}{x+1}+\dfrac{1}{x}+\dfrac{1}{x^2}$	½

3.		**Solve any FOUR of the following:**	**16**
	a)	Find the inverse of the matrix using adjoint method $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$	**04**
	Ans	Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$	
		$\lvert A \rvert = 1(24-25)-2(12-15)+3(10-12)$	
		$\therefore \lvert A \rvert = -1+6-6=-1$	½
		Matrix of minors $= \begin{bmatrix} \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} \end{bmatrix}$	
		$= \begin{bmatrix} -1 & -3 & -2 \\ -3 & -3 & -1 \\ -2 & -1 & 0 \end{bmatrix}$	1
		Matrix of cofactors $= \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$	1
		OR	
		$C_{11} = +\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24-25=-1$, $C_{12} = -\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = -(12-15)=3$, $C_{13} = +\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = 10-12=-2,$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	a)	$C_{21}=-\begin{vmatrix}2 & 3\\5 & 6\end{vmatrix}=-(12-15)=3, C_{22}=+\begin{vmatrix}1 & 3\\3 & 6\end{vmatrix}=6-9=-3,\ C_{23}=-\begin{vmatrix}1 & 2\\3 & 5\end{vmatrix}=-(5-6)=1$	
		$C_{31}=+\begin{vmatrix}2 & 3\\4 & 5\end{vmatrix}=(10-12)=-2,\ C_{32}=-\begin{vmatrix}1 & 3\\2 & 5\end{vmatrix}=-(5-6)=1, C_{33}=+\begin{vmatrix}1 & 2\\2 & 4\end{vmatrix}=4-4=0$	
		Matrix of cofactors $=\begin{bmatrix}-1 & 3 & -2\\3 & -3 & 1\\-2 & 1 & 0\end{bmatrix}$	2
		$\text{Adj}.A=\begin{bmatrix}-1 & 3 & -2\\3 & -3 & 1\\-2 & 1 & 0\end{bmatrix}$	1
		$A^{-1}=\frac{1}{\lvert A\rvert}\text{Adj}.A$	
		$\therefore A^{-1}=\frac{1}{-1}\begin{bmatrix}-1 & 3 & -2\\3 & -3 & 1\\-2 & 1 & 0\end{bmatrix}$	½
		$\therefore A^{-1}=\begin{bmatrix}1 & -3 & 2\\-3 & 3 & -1\\2 & -1 & 0\end{bmatrix}$	
		---	
	b)	Resolve into partial fraction : $\frac{x+2}{(x-1)(x^2+x+1)}$	**04**
	Ans	Let $\frac{x+2}{(x-1)(x^2+x+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$	½
		$\therefore x+2=A\left(x^2+x+1\right)+\left(Bx+C\right)\left(x-1\right)$	
		Put $x=1$	
		$\therefore 3=3A$	
		$\therefore A=1$	1
		Put $x=0$	
		$2=(1)A+(-1)C$	
		$2=(1)(1)-C$	

Q. No.	Sub Q. N.	Answers	Marking Scheme
3.	b)	$\therefore C=-1$	1
		Put $x=-1$	
		$\therefore -1+2=(1)A+(-2)(-B+C)$	
		$\therefore 1=A+2B-2C$	
		$\therefore 1=1+2B-2(-1)$	
		$\therefore 1-3=2B$	
		$\therefore -2=2B$	
		$\therefore B=-1$	1
		$\therefore \frac{x+2}{(x-1)(x^2+x+1)}=\frac{1}{x-1}+\frac{-x-1}{x^2+x+1}$	½
		$\therefore \frac{x+2}{(x-1)(x^2+x+1)}=\frac{1}{x-1}-\frac{x+1}{x^2+x+1}$	
		---	
	c)	Resolve into partial fraction : $\frac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}$	**04**
	Ans	$\frac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}$	
		Put $\tan\theta=y$	
		$\therefore \frac{y+1}{(y+2)(y+3)}$	½
		$\therefore \frac{y+1}{(y+2)(y+3)}=\frac{A}{y+2}+\frac{B}{y+3}$	½
		$\therefore \ y+1=A(y+3)+B(y+2)$	
		Put $y=-2$	
		$\therefore \ -1=A(-2+3)$	
		$\therefore A=-1$	1
		Put $y=-3$	
		$\therefore \ -3+1=B(-3+2)$	
		$\therefore B=2$	1
		$\therefore \frac{y+1}{(y+2)(y+3)}=\frac{-1}{y+2}+\frac{2}{y+3}$	½
		Replace y by $\tan\theta$	

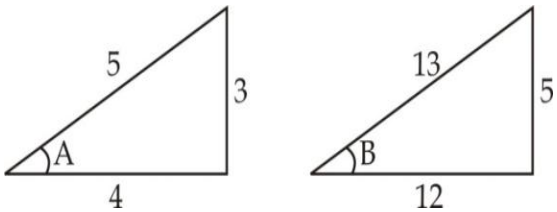
Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	$\therefore \dfrac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)} = \dfrac{-1}{\tan\theta+2} + \dfrac{2}{\tan\theta+3}$	½
	d) Ans	Prove that $\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$ Consider the unit circle with centre at origin. Let A be the angle P_4OP_1 and B be the angle P_1OP_2. Then $(A+B)$ is the angle P_4OP_2.Also let $(-B)$ be the angle of P_4OP_3. $\therefore P_1[\cos A, \sin A], P_2[\cos(A+B), \sin(A+B)], P_3[\cos(-B), \sin(-B)]$ and $P_4[1,0]$	**04**
			
		$\therefore P_1P_3 = P_2P_4$ ---------- $(\therefore \Delta P_1OP_3 \cong P_2OP_4)$ $\therefore \sqrt{[\cos A-\cos(-B)]^2+[\sin A-\sin(-B)]^2} = \sqrt{[1-\cos(A+B)]^2+[0-\sin(A+B)]^2}$ $\therefore [\cos A-\cos B]^2-[\sin A+\sin B]^2 = [1-\cos(A+B)]^2+[0-\sin(A+B)]^2$ $\therefore \cos^2 A-2\cos A\cos B+\cos^2 B+\sin^2 A+2\sin A\sin B+\sin^2 B$ $=1-2\cos(A+B)+\cos^2(A+B)+\sin^2(A+B)$	1
		$\therefore 2-2(\cos A\cos B-\sin A\sin B) = 2-2\cos(A+B)$ $\therefore \cos A\cos B-\sin A\sin B = \cos(A+B)$ $\cos(A+B) = \cos A\cos B-\sin A\sin B$	1
		Replace A by $\left(\dfrac{\pi}{2}+A\right)$ $\therefore \cos\left(\left(\dfrac{\pi}{2}+A\right)+B\right) = \cos\left(\dfrac{\pi}{2}+A\right)\cos B-\sin\left(\dfrac{\pi}{2}+A\right)\sin B$ $\therefore \cos\left(\dfrac{\pi}{2}+(A+B)\right) = -\sin A\cos B-\cos A\sin B$	1

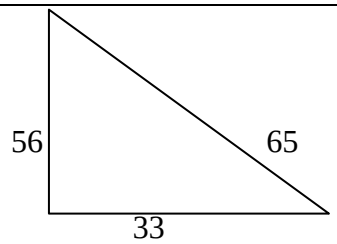
Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$\therefore -\sin(A+B) = -(\sin A\cos B + \cos A\sin B)$ $\therefore \sin(A+B) = \sin A\cos B + \cos A\sin B$	1
	e)	Prove that $\dfrac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A} = \tan 5A$	**04**
	Ans	$LHS = \dfrac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A}$ $= \dfrac{\sin 6A + \sin 4A + \sin 5A}{\cos 6A + \cos 4A + \cos 5A}$ $= \dfrac{2\sin\left(\dfrac{6A+4A}{2}\right).\cos\left(\dfrac{6A-4A}{2}\right) + \sin 5A}{2\cos\left(\dfrac{6A+4A}{2}\right).\cos\left(\dfrac{6A-4A}{2}\right) + \cos 5A}$ $= \dfrac{2\sin 5A\cdot\cos A + \sin 5A}{2co5A\cdot\cos A + \cos 5A}$ $= \dfrac{\sin 5A[2\cos A + 1]}{\cos 5A[2\cos A + 1]}$ $= \dfrac{\sin 5A}{\cos 5A}$ $= \tan 5A = R.H.S$	2 ½ ½ ½ ½
	f)	Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$	**04**
	Ans	$L.H.S. = \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$ $= \pi + \tan^{-1}\left(\dfrac{1+2}{1-(1\times 2)}\right) + \tan^{-1}(3)$ $= \pi + \tan^{-1}\left(\dfrac{3}{-1}\right) + \tan^{-1}(3)$ $= \pi + \tan^{-1}(-3) + \tan^{-1}(3)$ $= \pi - \tan^{-1}(3) + \tan^{-1}(3)$ $= \pi$ $= R.H.S.$	1 1 1 1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.		**Solve any FOUR of the following:**	**16**
	a)	Prove that $1+\tan A\cdot\tan 2A=\sec 2A$	**04**
	Ans	$LHS=1+\tan A\cdot\tan 2A$	
		$=1+\frac{\sin A}{\cos A}\cdot\frac{\sin 2A}{\cos 2A}$	
		$=\frac{\cos 2A\cdot\cos A+\sin 2A\cdot\sin A}{\cos 2A\cdot\cos A}$	1
		$=\frac{\cos(2A-A)}{\cos 2A\cdot\cos A}$	1
		$=\frac{\cos A}{\cos 2A\cdot\cos A}$	1
		$=\frac{1}{\cos 2A}$	
		$=\sec 2A$	1
		$=R.H.S.$	
		---	
	b)	Prove that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8A}}}=2\cos A$	**04**
	Ans	$LHS=\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8A}}}$	
		$=\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8A)}}}$	
		$=\sqrt{2+\sqrt{2+\sqrt{2\cdot 2\cos^2 4A}}}$	1
		$=\sqrt{2+\sqrt{2+2\cos 4A}}$	½
		$=\sqrt{2+\sqrt{2(1+\cos 4A)}}$	
		$=\sqrt{2+\sqrt{2\cdot 2\cos^2 2A}}$	1
		$=\sqrt{2+2\cdot\cos 2A}$	½
		$=\sqrt{2(1+\cos 2A)}$	
		$=\sqrt{2\cdot 2\cos^2 A}$	1
		$=2\cos A$	
		$=R.H.S$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	Without using calculator find the value of $\sin 420^0 \cdot \cos 390^0 + \cos(-300^0) \cdot \sin 330^0$	**04**
	Ans	$\sin 420^0 \cdot \cos 390^0 + \cos 300^0 \cdot \sin 330^0$	
		$= \sin(360^0 + 60^0) \cdot \cos(360^0 + 30^0) + \cos(360^0 - 60^0)\sin(360^0 - 30^0)$	1
		$= \sin 60^0 \cdot \cos 30^0 + \cos 60^0 \cdot (-\sin 30^0)$	1
		$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2}$	1
		$= \frac{3}{4} - \frac{1}{4}$	
		$= \frac{1}{2}$	1
	d)	If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, find $\tan(A+B)$	**04**
	Ans	$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$	
		$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}$	2
		$= \frac{5/6}{5/6}$	
		$= 1$	2
	e)	Find the principal value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}(\infty)$	**04**
	Ans	$\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}(\infty)$	
		$= \sin^{-1}\left(\frac{1}{2}\right) + \pi - \cos^{-1}\left(\frac{1}{2}\right) - \tan^{-1}(\infty)$	½
		$= \frac{\pi}{6} + \pi - \frac{\pi}{3} - \frac{\pi}{2}$ or $30^0 + 180^0 - 60^0 - 90^0$	1+1+1
		$= \frac{\pi}{3}$ or 60^0	½

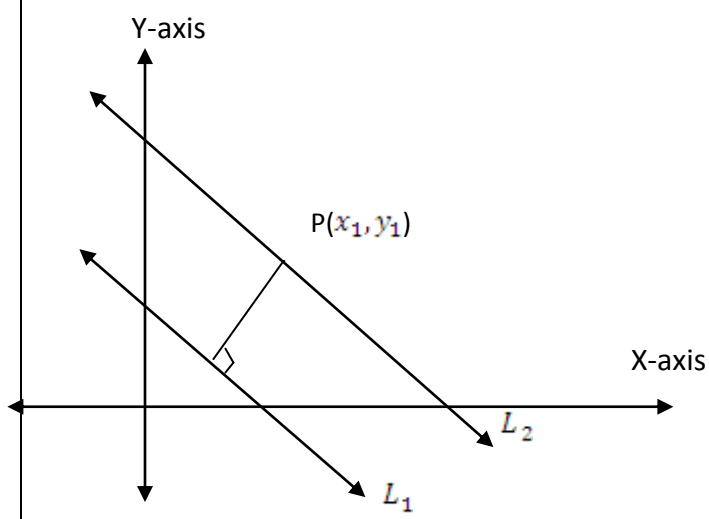
Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	f)	Prove that $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$	**04**
	Ans	$\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$	
		Let $\cos^{-1}\left(\frac{4}{5}\right)=A$	
		$\therefore \cos A=\frac{4}{5}$	
		$\therefore \sin^2 A=1-\cos^2 A$	
		$=1-\frac{16}{25}$	
		$\therefore \sin^2 A=\frac{9}{25}$	
		$\therefore \sin A=\frac{3}{5}$	1
		$\cos^{-1}\left(\frac{12}{13}\right)=B$	
		$\therefore \cos B=\frac{12}{13}$	
		$\therefore \sin^2 B=1-\cos^2 B$	
		$\therefore \sin^2 B=1-\frac{144}{169}$	
		$\therefore \sin^2 B=\frac{25}{169}$	
		$\therefore \sin B=\frac{5}{13}$	1
		$\therefore \cos(A+B)=\cos A\cos B-\sin A\sin B$	
		$=\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)-\left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$	
		$\therefore \cos(A+B)=\frac{33}{65}$	1
		$\therefore A+B=\cos^{-1}\left(\frac{33}{65}\right)$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	f)	$\therefore \cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$	1
		OR	
		Let $\cos^{-1}\left(\frac{4}{5}\right)=A$ $\therefore \cos A=\frac{4}{5}$ $\therefore \tan A=\frac{3}{4}$ $A=\tan^{-1}\left(\frac{3}{4}\right)$ 	
		$\therefore \cos^{-1}\left(\frac{4}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)$	1
		$\cos^{-1}\left(\frac{12}{13}\right)=B$ $\therefore \cos B=\frac{12}{13}$ $\therefore \tan B=\frac{5}{12}$ $B=\tan^{-1}\left(\frac{5}{12}\right)$	
		$\therefore \cos^{-1}\left(\frac{12}{13}\right)=\tan^{-1}\left(\frac{5}{12}\right)$	1
		$L.H.S.=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{5}{12}\right)$ $=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\frac{5}{12}}\right)$ $=\tan^{-1}\left(\frac{56}{33}\right)$	1

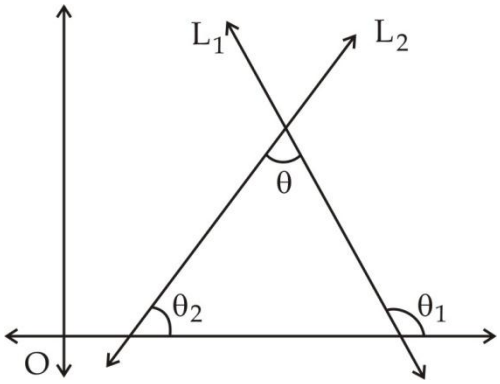
Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	f)	Let $\tan^{-1}\left(\frac{56}{33}\right)=C$ $\therefore \tan C=\frac{56}{33}$ $\therefore \cos C=\frac{33}{65}$ $\therefore C=\cos^{-1}\left(\frac{33}{65}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$ 	1
5.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Prove that $\cos 20^0\cdot\cos 40^0\cdot\cos 60^0\cdot\cos 80^0=\frac{1}{16}$	**04**
	Ans	$L.H.S.=\cos 20^0\cdot\cos 40^0\cdot\cos 60^0\cdot\cos 80^0$	
		$=\frac{1}{2}\left(2\cos 40^0\cdot cos 20^0\right)\cdot\frac{1}{2}\cdot\cos 80^0$	½
		$=\frac{1}{4}\left[\cos(40^0+20^0)+\cos(40^0-20^0)\right]\cos 80^0$	
		$=\frac{1}{4}\cos 80^0\cdot\left[\cos 60^0+\cos 20^0\right]$	1
		$=\frac{1}{4}\cos 80^0\cdot\left[\frac{1}{2}+\cos 20^0\right]$	½
		$=\frac{1}{8}\cos 80^0+\frac{1}{4}\cos 80^0\cdot\cos 20^0$	
		$=\frac{1}{8}\cos 80^0+\frac{1}{8}\left[2\cos 80^0\cdot\cos 20^0\right]$	
		$=\frac{1}{8}\cos 80^0+\frac{1}{8}\left[\cos(80^0+20^0)+\cos(80^0-20^0)\right]$	1
		$=\frac{1}{8}\cos 80^0+\frac{1}{8}\left[\cos 100^0+\cos 60^0\right]$	
		$=\frac{1}{8}\cos 80^0+\frac{1}{8}\left[\cos(180^0-80^0)+\frac{1}{2}\right]$	½

Q. No.	Sub Q. N.	Answers	Marking Scheme
5.	a)	$L.H.S. = \frac{1}{8}\cos 80^0 + \frac{1}{8}\left[-\cos 80^0 + \frac{1}{2}\right]$	
		$= \frac{1}{8}\cos 80^0 - \frac{1}{8}\cos 80^0 + \frac{1}{16}$	
		$= \frac{1}{16}$	½
		$= R.H.S.$	
	b)	Prove that $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $x > 0, y > 0$ and $xy < 1$.	**04**
	Ans	Let $\tan^{-1} x = A$ and $\tan^{-1} y = B$ $\therefore x = \tan A$ $\therefore y = \tan B$	1
		$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	1
		$\therefore \tan(A+B) = \frac{x+y}{1-xy}$	½
		$\therefore A + B = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$	1
		$\therefore \tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$	½

Q. No.	Sub Q. N.	Answers	Marking Scheme
5.	c)	Prove that $\dfrac{\sin 4\theta+\sin 2\theta}{1+\cos 2\theta+\cos 4\theta}=\tan 2\theta$	**04**
	Ans	$L.H.S.=\dfrac{\sin 4\theta+\sin 2\theta}{1+\cos 2\theta+\cos 4\theta}$	
		$=\dfrac{2\sin 2\theta\cdot\cos 2\theta+\sin 2\theta}{1+\cos 4\theta+\cos 2\theta}$	1
		$=\dfrac{2\sin 2\theta\cdot\cos 2\theta+\sin 2\theta}{2\cos^2 2\theta+\cos 2\theta}$	1
		$=\dfrac{\sin 2\theta(2\cos 2\theta+1)}{\cos 2\theta(2\cos 2\theta+1)}$	1
		$=\dfrac{\sin 2\theta}{\cos 2\theta}$	
		$=\tan 2\theta$	1
		$=R.H.S.$	
	d)	Find the angle between two lines $y=5x+6$ and $y=x$	**04**
	Ans	For $y=5x+6 \Rightarrow 5x-y+6=0$	
		slope $m_1=-\dfrac{a}{b}=-\dfrac{-5}{-1}=5$	1
		For $y=x \Rightarrow x-y=0$	
		slope $m_2=-\dfrac{a}{b}=-\dfrac{-1}{-1}=1$	1
		$\tan\theta=\left\lvert\dfrac{m_1-m_2}{1+m_1m_2}\right\rvert$	
		$=\left\lvert\dfrac{5-1}{1+(5\times 1)}\right\rvert$	1
		$=\left\lvert\dfrac{4}{6}\right\rvert$	
		$\tan\theta=\dfrac{2}{3}$	
		$\therefore\ \theta=\tan^{-1}\left(\dfrac{2}{3}\right)$	1

Q. No.	Sub Q. N.	Answers	Marking Scheme
5.	e)	Prove that the distance between two parallel line $ax+by+c_1=0$ and $ax+by+c_2=0$ is $\left\|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right\|$	**04**
	Ans	Let $L_1 = ax+by+c_1 = 0$ and $L_2 = ax+by+c_2 = 0$	
		Let (x_1, y_1) be a point on the line L_2 $ax_1+by_1+c_2=0$	1
		$\therefore\ ax_1+by_1=-c_2$	
			1
		now the perpendicular distance from (x_1, y_1) on L_1 is	
		$p=\left\|\frac{ax_1+by_1+c_1}{\sqrt{a^2+b^2}}\right\|$	1
		$=\left\|\frac{-c_2+c_1}{\sqrt{a^2+b^2}}\right\|$	1
		$\therefore\ p=\left\|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right\|$	
		OR	
		$p=\left\|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right\|$	

Q. No.	Sub Q. N.	Answers	Marking Scheme
5.	f)	Find the equation of a line passing through the point of intersection of the lines $2x+3y=13$, $5x-y=7$ and perpendicular to the line $3x-y+7=0$.	**04**
	Ans	$2x+3y=13$, $5x-y=7$	
		$\therefore \quad 2x+3y=13$	
		$\quad 15x-3y=21$	
		$\therefore \quad 17x=34 \Rightarrow x=2$	
		$\therefore \quad y=3$	
		$\therefore$ point of intersection $=(2,3)=(x_1,y_1)$	2
		and slope of given line is $m_1=-\frac{a}{b}=\frac{-3}{-1}=3$	½
		$\therefore$ slope of required line is $m_2=\frac{-1}{m_1}=\frac{-1}{3}$	½
		$\therefore$ Equation of line is $y-y_1=m(x-x_1)$	
		$\therefore \; y-3=\frac{-1}{3}(x-2)$	½
		$\therefore \; 3(y-3)=-1(x-2)$	
		$\therefore \; 3y-9=-x+2$	
		$\therefore \; x+3y-11=0$	½

6.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	If m_1 and m_2 are slopes of any two lines L_1 and L_2 then prove that angle between two lines L_1 and L_2 is $\theta=\tan^{-1}\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$	**04**
	Ans		1

Q. No.	Sub Q. N.	Answers	Marking Scheme
6.	a)	Let θ_1 = Angle of inclination of L_1 θ_2 = Angle of inclination of L_2 $\therefore$ Slope of L_1 is $m_1 = \tan\theta_1$ Slope of L_2 is $m_2 = \tan\theta_2$ from figure, $\theta = \theta_1 - \theta_2$ $\therefore \tan\theta = \tan(\theta_1 - \theta_2)$ $\therefore \tan\theta = \dfrac{\tan(\theta_1) - \tan(\theta_2)}{1 + \tan(\theta_1)\tan(\theta_2)}$ $\therefore \tan\theta = \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}$ $\because \theta$ is acute $\therefore \tan\theta = \left\lvert \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2} \right\rvert$ $\therefore \theta = \tan^{-1}\left\lvert \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2} \right\rvert$	½ ½ 1 ½ ½
	b)	Find the equation of a line passing through the point (6,5) and parallel to the line having intercepts 2 and 4 on X and Y axis respectively.	**04**
	Ans	Equation of line is $\dfrac{x}{a} + \dfrac{y}{b} = 1$ $\therefore \dfrac{x}{2} + \dfrac{y}{4} = 1$ $\therefore 4x + 2y = 8$ $\therefore 2x + y = 4$ Slope of the line $2x + y = 4$ *is* $m_1 = -\dfrac{a}{b} = \dfrac{-2}{1} = -2$ Slope of required line is $m_2 = m_1 = -2$ The equation of line is $y - y_1 = m(x - x_1)$ $\therefore y - 5 = -2(x - 6)$	1 1 1

Q. No.	Sub Q. N.	Answers	Marking Scheme
6.	b)	$\therefore y-5=-2x+12$ $\therefore 2x+y-17=0$	1
	c)	Two sets of observations are given below:	**04**

Set-I	Set-II
$\bar{x}=82.5$	$\bar{x}=48.75$
$\sigma=7.3$	$\sigma=8.35$

Which set is more consistent?

Ans

Coefficient of variation $V=\frac{\sigma}{\bar{x}}\times 100$

For Set-I

Coefficient of variation $V_1=\frac{7.3}{82.5}\times 100$

$\therefore V_1=8.848$ — $1\frac{1}{2}$

For Set-II

Coefficient of variation $V_2=\frac{8.35}{48.75}\times 100$

$\therefore V_2=17.128$ — $1\frac{1}{2}$

$\therefore V_1<V_2$

$\therefore$ Set-I is more consistent — 1

d) Find mean deviation from mean for the following data — **04**

Marks Obtained	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	4	6	10	18	9	3

Q. No.	Sub Q. N.	Answers	Marking Scheme
6.	d) Ans		

C.I.	f_i	x_i	$f_i x_i$	$d_i = \lvert x_i - \bar{x} \rvert$	$f_i d_i$
10-20	4	15	60	26.2	104.8
20-30	6	25	150	16.2	97.2
30-40	10	35	350	6.2	62
40-50	18	45	810	3.8	68.4
50-60	9	55	495	13.8	124.2
60-70	3	65	195	23.8	71.4
	$\sum f_i = 50$		$\sum f_i x_i = 2060$		$\sum f_i d_i = 528$

2

$$Mean\ \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\therefore \bar{x} = \frac{2060}{50}$$

$$\therefore \bar{x} = 41.2$$

1

$$M.D = \frac{\sum f_i d_i}{f_i}$$

$$\therefore M.D = \frac{528}{50}$$

$$\therefore M.D. = 10.56$$

1

---

e) Find the standard deviation from following: **04**

Class interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	3	5	9	15	20	16	10	2

Q. No.	Sub Q. N.	Answers	Marking Scheme
6.	e) Ans		

C.I.	x_i	f_i	u_i	f_iu_i	u_i^2	$f_iu_i^2$
0-5	2.5	3	-3	-9	9	27
5-10	7.5	5	-2	-10	4	20
10-15	12.5	9	-1	-9	1	9
15-20	17.5	15	0	0	0	0
20-25	22.5	20	1	20	1	20
25-30	27.5	16	2	32	4	64
30-35	32.5	10	3	30	9	90
35-40	37.5	2	4	8	16	32
		$N=80$		$\sum f_iu_i = 62$		$\sum f_iu^2{}_i = 262$

2

$$\text{S.D.}\,\sigma = \sqrt{\frac{\sum f_iu_i^2}{\sum f_i} - \left(\frac{\sum f_iu_i}{\sum f_i}\right)^2} \times c$$

$$= \sqrt{\frac{262}{80} - \left(\frac{62}{80}\right)^2} \times 5$$ 1

$$= 8.177$$ 1

---

f) Find the variance and co-efficient of variance for the following **04**

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	14	23	27	21	15

Q. No.	Sub Q. N.	Answers	Marking Scheme
6.	f) Ans		

Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	14	70	25	350
10-20	15	23	345	225	5175
20-30	25	27	675	625	16875
30-40	35	21	735	1225	25725
40-50	45	15	675	2025	30375
Total		$\sum f_i = 100$	$\sum f_i x_i = 2500$		78500

2

Mean $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{2500}{100} = 25$ ½

S.D. $\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$

$= \sqrt{\frac{78500}{100} - (25)^2}$ ½

$\sigma = 12.649$

Variance $= \sigma^2 = (12.649)^2 = 159.99 \approx 160$ ½

Coeefficient of variance $= \frac{\sigma}{\bar{x}} \times 100 = \frac{12.649}{25} \times 100 = 50.596 \approx 50.6$ ½

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.